Fundamentals Physics

Eleventh Edition

Halliday

Chapter 9

Center of Mass and Linear Momentum

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9-1 Center of Mass (1 of 14)

Learning Objectives

9.01 Given the positions of several particles along an axis or a plane, determine the location of their center of mass.

9.02 Locate the center of mass of an extended, symmetric object by using the symmetry.

9.03 For a two-dimensional or three-dimensional extended object with a uniform distribution of mass, determine the center of mass by (a) mentally dividing the object into simple geometric figures, each of which can be replaced by a particle at its center and (b) finding the center of mass of those particles.

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9-1 Center of Mass (2 of 14)

- The motion of rotating objects can be complicated (imagine flipping a baseball bat into the air)
- But there is a special point on the object for which the motion is simple
- The center of mass of the bat traces out a parabola, just as a tossed ball does
- All other points rotate around this point

Figure 9-1

9-1 Center of Mass (3 of 14)

- The **center of mass** (com) of a system of particles:

  The center of mass of a system of particles is the point that moves as though (1) all of the system's mass were concentrated there and (2) all external forces were applied there.

- For two particles, for an arbitrary choice of origin:

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$ **Equation (9-1)**

9-1 Center of Mass (5 of 14)

- The center of mass is in the same location regardless of the coordinate system used
- It is a property of the particles, not the coordinates

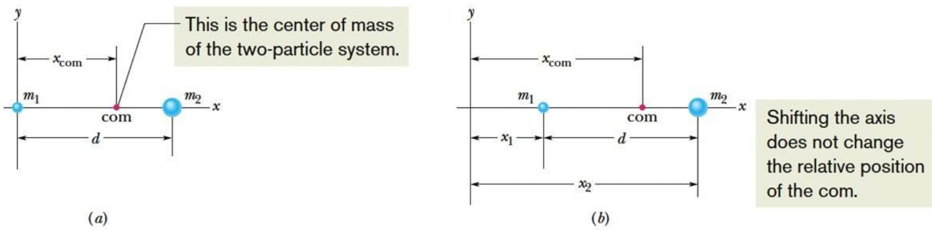

Figure 9-2

9-1 Center of Mass (6 of 14)

- For many particles, we can generalize the equation, where $M = m_1 + m_2 + \cdots + m_n$:

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{M} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i.$$

Equation (9-4)

- In three dimensions, we find the center of mass along each axis separately:

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \quad y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \quad z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i.$$

Equation (9-5)

9-1 Center of Mass (7 of 14)

- More concisely, we can write in terms of vectors:

$$\vec{r}_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i,$$

Equation (9-8)

- For solid bodies, we take the limit of an infinite sum of infinitely small particles → integration!
- Coordinate-by-coordinate, we write:

$$x_{\text{com}} = \frac{1}{M}\int x\ dm, \quad y_{\text{com}} = \frac{1}{M}\int y\ dm, \quad z_{\text{com}} = \frac{1}{M}\int z\ dm,$$

Equation (9-9)

- Here *M* is the mass of the object

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9-1 Center of Mass (8 of 14)

- We limit ourselves to objects of uniform density, ρ, for the sake of simplicity

$$\rho = \frac{dm}{dV} = \frac{M}{V},$$

Equation (9-10)

- Substituting, we find the center of mass simplifies:

$$x_{\text{com}} = \frac{1}{V}\int x\ dV, \quad y_{\text{com}} = \frac{1}{V}\int y\ dV, \quad z_{\text{com}} = \frac{1}{V}\int z\ dV.$$

Equation (9-11)

- You can bypass one or more of these integrals if the object has symmetry

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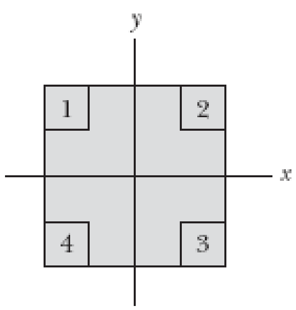
9-1 Center of Mass (9 of 14)

- The center of mass lies at a point of symmetry (if there is one)
- It lies on the line or plane of symmetry (if there is one)
- It need not be on the object (consider a doughnut)

9-1 Center of Mass (10 of 14)

Checkpoint 1

The figure shows a uniform square plate from which four identical squares at the corners will be removed. (a) Where is the center of mass of the plate originally? Where is it after the removal of (b) square 1; (c) squares 1 and 2; (d) squares 1 and 3; (e) squares 1, 2, and 3; (f) all four squares? Answer in terms of quadrants, axes, or points (without calculation, of course).



Answer:

(a) at the origin
(b) in Q_4, along $y = -x$
(c) along the $-y$ axis
(d) at the origin
(e) in Q_3, along $y = x$
(f) at the origin

9-1 Center of Mass (11 of 14)

Example Subtracting

- Task: find com of a disk with another disk taken out of it

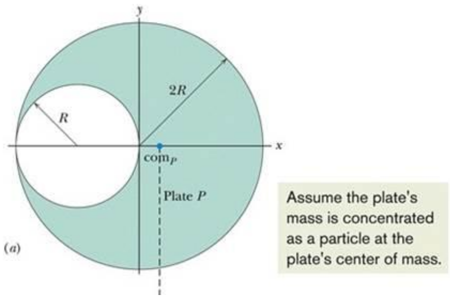


Figure 9-4

9-1 Center of Mass (12 of 14)

Example Subtracting

- Find the com of each individual disk (start from the bottom and work up)
- Find the com of the two individual coms (one for each disk), treating the cutout as having negative mass
- On the diagram, com_C is the center of mass for Plate P and Disk S combined
- com_P is the center of mass for the composite plate with Disk S removed

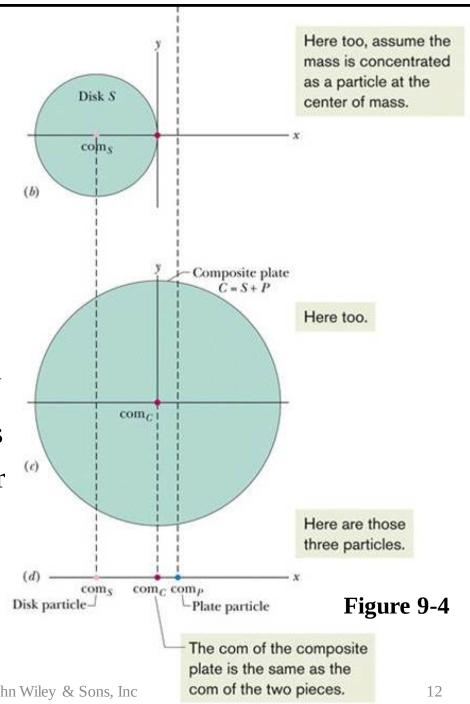


Figure 9-4

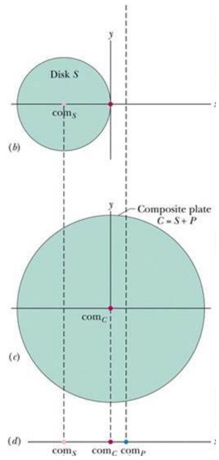
9-1 Center of Mass (13 of 14)

Plate	Center of Mass	Location of com	Mass
P	com_P	$x_P = ?$	m_P
S	com_S	$x_S = -R$	m_S
C	com_C	$x_C = 0$	$m_C = m_S + m_P$

Assume that mass m_S of disk S is concentrated in a particle at $x_S = -R$, and mass m_P is concentrated in a particle at x_P (Fig. 9-4*d*). Next we use Eq. 9-2 to find the center of mass x_{S+P} of the two-particle system:

$$x_{S+P} = \frac{m_S x_S + m_P x_P}{m_S + m_P}. \quad (9\text{-}12)$$

$$x_P = -x_S \frac{m_S}{m_P}. \quad (9\text{-}13)$$



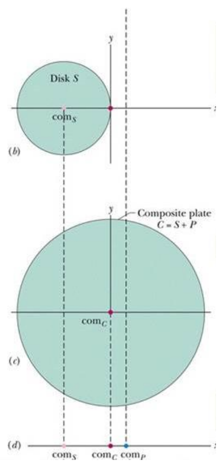
9-1 Center of Mass (14 of 14)

Because the plate is uniform, the densities and thicknesses are equal; we are left with

$$\frac{m_S}{m_P} = \frac{\text{area}_S}{\text{area}_P} = \frac{\text{area}_S}{\text{area}_C - \text{area}_S} = \frac{\pi R^2}{\pi(2R)^2 - \pi R^2} = \frac{1}{3}.$$

Substituting this and $x_S = -R$ into Eq. 9-13, we have

$$x_P = \tfrac{1}{3}R. \quad \text{(Answer)}$$



9-2 Newton's Second Law for a System of Particles (1 of 6)

Learning Objectives

9.04 Apply Newton's second law to a system of particles by relating the net force (of the forces acting on the particles) to the acceleration of the system's center of mass.

9.05 Apply the constant-acceleration equations to the motion of the individual particles in a system and to the motion of the system's center of mass.

9.06 Given the mass and velocity of the particles in a system, calculate the velocity of the system's center of mass.

9-2 Newton's Second Law for a System of Particles (2 of 6)

9.07 Given the mass and acceleration of the particles in a system, calculate the acceleration of the system's center of mass.

9.08 Given the position of a system's center of mass as a function of time, determine the velocity of the center of mass.

9.09 Given the velocity of a system's center of mass as a function of time, determine the acceleration of the center of mass.

9-2 Newton's Second Law for a System of Particles (3 of 6)

9.10 Calculate the change in the velocity of a com by integrating the com's acceleration function with respect to time.

9.11 Calculate a com's displacement by integrating the com's velocity function with respect to time.

9.12 When the particles in a two-particle system move without the system's com moving, relate the displacements of the particles and the velocities of the particles.

9-2 Newton's Second Law for a System of Particles (4 of 6)

- Center of mass motion continues unaffected by forces internal to a system (collisions between billiard balls)
- Motion of a system's center of mass:

$$\vec{F}_{net} = M\vec{a}_{com} \quad \text{(system of particles).}$$ **Equation (9-14)**

$$F_{net,x} = Ma_{com,x} \quad F_{net,y} = Ma_{com,y} \quad F_{net,z} = Ma_{com,z}.$$ **Equation (9-15)**

- Reminders:
  1. F_{net} is the sum of all external forces
  2. M is the total, constant, mass of the **closed** system
  3. a_{com} is the center of mass acceleration

9-2 Newton's Second Law for a System of Particles (5 of 6)

Examples Using the center of mass motion equation:

- Billiard collision: forces are only internal, $F = 0$ so $a = 0$
- Baseball bat: $a = g$, so com follows gravitational trajectory
- Exploding rocket: explosion forces are internal, so only the gravitational force acts on the system, and the com follows a gravitational trajectory as long as air resistance can be ignored for the fragments.

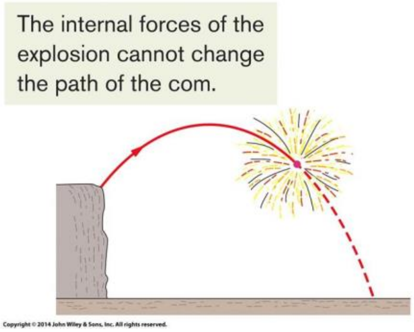
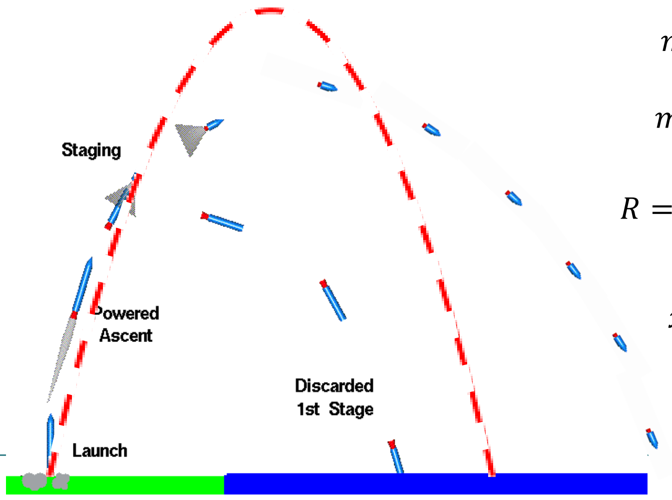


Figure 9-5

9-2 Newton's Second Law for a System of Particles (6 of 6)

Examples Using the center of mass motion equation



$$m_{booster} = 0.8M$$

$$m_{warhead} = 0.2M$$

$$R = \frac{m_b x_b + m_h x_h}{M}$$

$$x_b = \frac{R - 0.2x_h}{0.8}$$

9-3 Linear Momentum (1 of 6)

Learning Objectives

9.13 Identify that momentum is a vector quantity and thus has both magnitude and direction and also components.

9.14 Calculate the (linear) momentum of a particle as the product of the particle's mass and velocity.

9.15 Calculate the change in momentum (magnitude and direction) when a particle changes its speed and direction of travel.

9-3 Linear Momentum (2 of 6)

9.16 Apply the relationship between a particle's momentum and the (net) force acting on the particle.

9.17 Calculate the momentum of a system of particles as the product of the system's total mass and its center-of-mass velocity.

9.18 Apply the relationship between a system's center-of-mass momentum and the net force acting on the system.

9-3 Linear Momentum (3 of 6)

- The **linear momentum** is defined as:

$$\vec{p} = m\vec{v}$$ **Equation (9-22)**

- The momentum:
  - Points in the same direction as the velocity
  - Can only be changed by a net external force

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

- We can write Newton's second law thus:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$ **Equation (9-23)**

9-3 Linear Momentum (4 of 6)

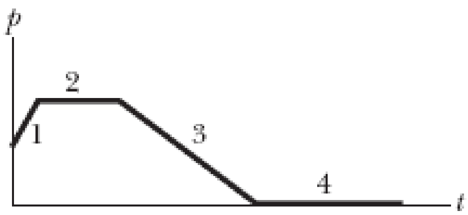
Checkpoint 3

The figure gives the magnitude *p* of the linear momentum versus time *t* for a particle moving along an axis. A force directed along the axis acts on the particle. (a) Rank the four regions indicated according to the magnitude of the force, greatest first. (b) In which region is the particle slowing?

Answer:

(a) 1, 3, 2 & 4

(b) region 3



9-3 Linear Momentum (5 of 6)

- We can sum momenta for a system of particles to find:

$$\vec{P} = M\vec{v}_{com} \quad \text{(linear momentum, system of particles),}$$

Equation (9-25)

The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

9-3 Linear Momentum (6 of 6)

- Taking the time derivative, we can write Newton's second law for a system of particles as:

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} \quad \text{(system of particles),}$$ **Equation (9-27)**

- The net external force on a system changes linear momentum
- Without a net external force, the total linear momentum of a system of particles cannot change

9-4 Collision and Impulse (1 of 12)

Learning Objectives

9.19 Identify that impulse is a vector quantity and thus has both magnitude and direction and components.

9.20 Apply the relationship between impulse and momentum change.

9.21 Apply the relationship between impulse, average force, and the time interval taken by the impulse.

9.22 Apply the constant-acceleration equations to relate impulse to force.

9-4 Collision and Impulse (2 of 12)

9.23 Given force as a function of time, calculate the impulse (and thus also the momentum change) by integrating the function.

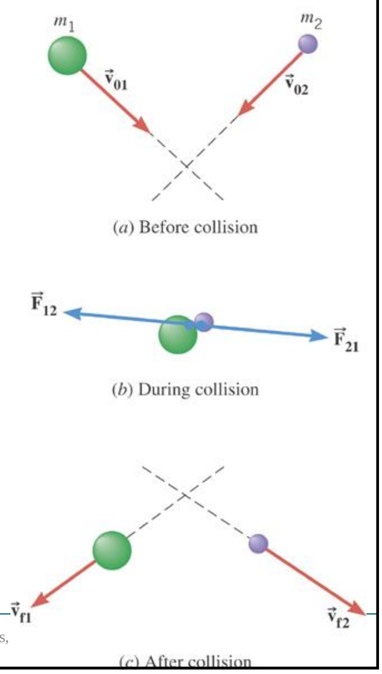
9.24 Given a graph of force versus time, calculate the impulse (and thus also the momentum change) by graphical integration.

9.25 In a continuous series of collisions by projectiles, calculate average force on the target by relating it to the mass collision rate and the velocity change of each projectile.

9-4 Collision and Impulse (3 of 12)

Internal forces – Forces that objects within the system exert on each other.

External forces – Forces exerted on objects by agents external to the system.



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9-4 Collision and Impulse (4 of 12)

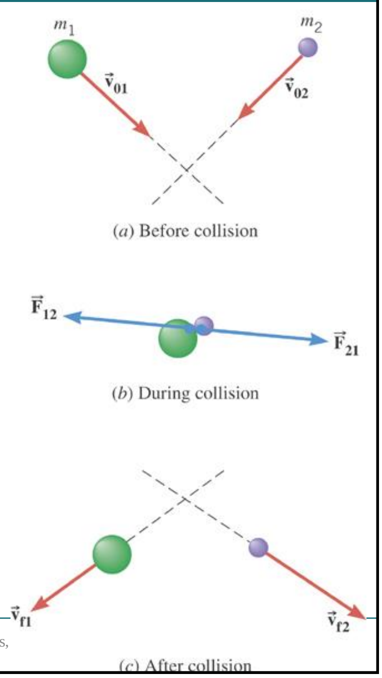
$$\left(\sum \vec{\mathbf{F}}\right)\Delta t = m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{o}}$$

OBJECT 1

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{F}}_{12}\right)\Delta t = m_1\vec{\mathbf{v}}_{\mathbf{f1}} - m_1\vec{\mathbf{v}}_{\mathbf{o1}}$$

OBJECT 2

$$\left(\vec{\mathbf{W}}_2 + \vec{\mathbf{F}}_{21}\right)\Delta t = m_2\vec{\mathbf{v}}_{\mathbf{f2}} - m_2\vec{\mathbf{v}}_{\mathbf{o2}}$$



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9-4 Collision and Impulse (5 of 12)

$$\left(\vec{\mathbf{W}}_1 + \vec{\bar{\mathbf{F}}}_{12}\right)\Delta t = m_1\vec{\mathbf{v}}_{\mathbf{f}1} - m_1\vec{\mathbf{v}}_{\mathbf{o}1}$$

$$+$$

$$\left(\vec{\mathbf{W}}_2 + \vec{\bar{\mathbf{F}}}_{21}\right)\Delta t = m_2\vec{\mathbf{v}}_{\mathbf{f}2} - m_2\vec{\mathbf{v}}_{\mathbf{o}2}$$

$$\Downarrow$$

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2 + \vec{\bar{\mathbf{F}}}_{12} + \vec{\bar{\mathbf{F}}}_{21}\right)\Delta t = (m_1\vec{\mathbf{v}}_{\mathbf{f}1} + m_2\vec{\mathbf{v}}_{\mathbf{f}2}) - (m_1\vec{\mathbf{v}}_{\mathbf{o}1} + m_2\vec{\mathbf{v}}_{\mathbf{o}2})$$

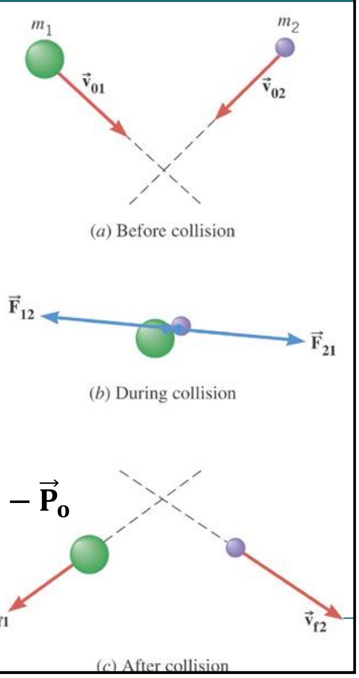
$\vec{\bar{\mathbf{F}}}_{12} = -\vec{\bar{\mathbf{F}}}_{21}$ $\qquad$ $\vec{\mathbf{P}}_{\mathbf{f}}$ $\qquad$ $\vec{\mathbf{P}}_{\mathbf{o}}$

9-4 Collision and Impulse (6 of 12)

The internal forces cancel out.

$$(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2)\Delta t = \vec{\mathbf{P}}_{\mathbf{f}} - \vec{\mathbf{P}}_{\mathbf{o}}$$

$$(\text{sum of average external forces})\Delta t = \vec{\mathbf{P}}_{\mathbf{f}} - \vec{\mathbf{P}}_{\mathbf{o}}$$



9-4 Collision and Impulse (7 of 12)

- In a collision, momentum of a particle can change
- We define the **impulse *J*** acting during a collision:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\ dt$$ **Equation (9-30)**

- This means that the applied impulse is equal to the change in momentum of the object during the collision

$$\Delta\vec{p} = \vec{J} \quad \text{(linear momentum−impulse theorem).}$$

Equation (9-31)

- This equation can be rewritten component-by-component, like other vector equations

9-4 Collision and Impulse (8 of 12)

- Given F_{avg} and duration:

$$J = F_{\text{avg}}\Delta t.$$ **Equation (9-35)**

- We are integrating, we only need to know the area under the force curve

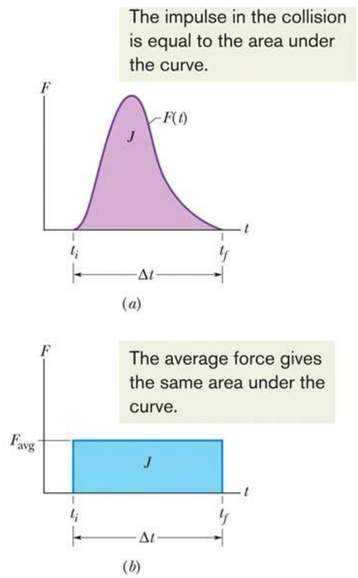


Figure 9-9

9-4 Collision and Impulse (9 of 12)

Checkpoint 4

A paratrooper whose chute fails to open lands in snow; he is hurt slightly. Had he landed on bare ground, the stopping time would have been 10 times shorter and the collision lethal. Does the presence of the snow increase, decrease, or leave unchanged the values of (a) the paratrooper's change in momentum, (b) the impulse stopping the paratrooper, and (c) the force stopping the paratrooper?

Answer:

(a) unchanged

(b) unchanged

(c) decreased

9-4 Collision and Impulse (10 of 12)

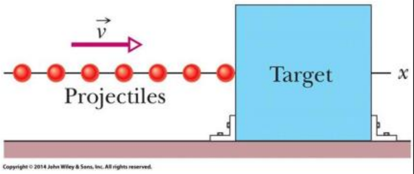


Figure 9-10

- For a steady stream of n projectiles, each undergoes a momentum change Δp

$$J = -n\Delta p,$$ **Equation (9-36)**

- The average force is:

$$F_{avg} = \frac{J}{\Delta t} = -\frac{n}{\Delta t}\Delta p = -\frac{n}{\Delta t} m\, \Delta v.$$ **Equation (9-37)**

- If the particles stop:

$$\Delta v = v_f - v_i = 0 - v = -v,$$ **Equation (9-38)**

- If the particles bounce back with equal speed:

$$\Delta v = v_f - v_i = -v - v = -2v.$$ **Equation (9-39)**

9-4 Collision and Impulse (11 of 12)

- The product *nm* is the total mass for *n* collisions so we can write:

$$F_{avg} = -\frac{\Delta m}{\Delta t}\Delta v. \qquad \text{Equation (9-40)}$$

9-4 Collision and Impulse (12 of 12)

Checkpoint 5

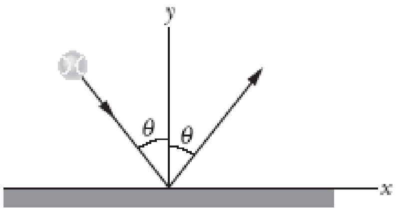
The figure shows an overhead view of a ball bouncing from a vertical wall without any change in its speed. Consider the change $\Delta\vec{p}$ in the ball's linear momentum. (a) Is Δp_x positive, negative, or zero? (b) Is Δp_y positive, negative, or zero? (c) What is the direction of $\Delta\vec{p}$?

Answer:

(a) zero

(b) positive

(c) along the positive y-axis (normal force)



9-5 Conservation of Linear Momentum (1 of 12)

Learning Objectives

9.26 For an isolated system of particles, apply the conservation of linear momenta to relate the initial momenta of the particles to their momenta at a later instant.

9.27 Identify that the conservation of linear momentum can be done along an individual axis by using components along that axis, provided there is no net external force component along that axis.

9-5 Conservation of Linear Momentum (2 of 12)

- For an impulse of zero we find:

$$\vec{P} = \text{constant} \quad \text{(closed, isolated system).}$$ **Equation (9-42)**

- Which says that:

If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ of the system cannot change.

9-5 Conservation of Linear Momentum (3 of 12)

- This is called the **law of conservation of linear momentum**
- Check the components of the net external force to know if you should apply this

If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

9-5 Conservation of Linear Momentum (4 of 12)

Both are at rest when fired, no net external force

Total Momentum: $\vec{P}_{initial} = \vec{P}_{final} = 0$

What does this say about the motion after the gun is fired?

9-5 Conservation of Linear Momentum (5 of 12)

$$\vec{P}_{initial} = \vec{P}_{final} = 0$$

$$\vec{P}_{final} = m_b\vec{v}_b + m_g\vec{v}_g = 0$$

$$\vec{v}_g = -\frac{m_b}{m_g}\vec{v}_b$$

$m_g >> m_b$ so $v_g << v_b$

The gun is heavier so it moves slower!

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9-5 Conservation of Linear Momentum (6 of 12)

Does the Center of Mass of the system move?

$$\vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{d}{dt}\left[\frac{m_b\vec{r}_b + m_g\vec{r}_g}{m_g + m_g}\right] = \frac{m_b\vec{v}_b + m_g\vec{v}_g}{m_g + m_g}$$

No, because $P_{initial} = P_{final} = 0$ for the system!

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9-5 Conservation of Linear Momentum (7 of 12)

$\vec{v}_G$ ←  → $\vec{v}_B$

If the momenta are the same, why do you want to hold the gun rather than catch the bullet?

Consider the kinetic energy:

$$K = \tfrac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$|p_g| = |pb| \text{ and } mb << mg \rightarrow K_b >> K_g$$

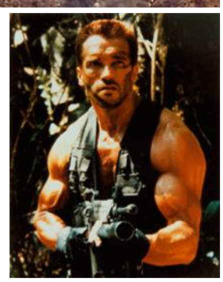
9-5 Conservation of Linear Momentum (8 of 12)

Arnold: 300 lbs ~136 kg

Thrown back ~ 3m in 1s ⇒ 3 m/s

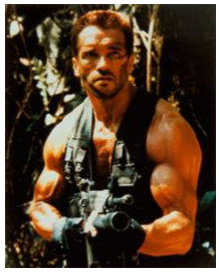
← P_{Arnold} → P_{bullet}

$$p_{Arnold} + p_{bullet} = 0$$
$$p_{Arnold} = -p_{bullet}$$



$$p_{Arnold} = m_{Arnold}v_{Arnold}$$
$$= (136kg)(-3m/s) = -408kgm$$
$$p_{bullet} = 408kgm/s$$

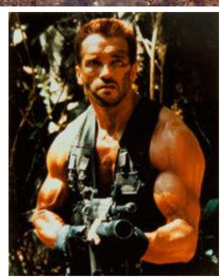
9-5 Conservation of Linear Momentum (9 of 12)



So how much energy is this? (Assume $m_{bullet} = 5$ gr)

$$K = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$
$$= \frac{408^2}{10^{-2}} = 16.67 \times 10^6 \ J$$

9-5 Conservation of Linear Momentum (10 of 12)



And hence, the velocity of the bullet is

$$v_{bullet} = \sqrt{\frac{2K}{m}} = \sqrt{\frac{3.33 \times 10^7}{5 \times 10^{-3}}}$$
$$= 8.16 \times 10^4 \ \frac{m}{s} = 81.6 \ \frac{km}{s}$$

or

$$v_{bullet} = \frac{p_{bullet}}{m_{bullet}} = \frac{408}{0.005} = 81.6 km/s$$

This is very-very large value !!!

Typical speed of a bullet is $975 \frac{m}{s}$ (M16 rifle)

9-5 Conservation of Linear Momentum (11 of 12)

- Internal forces can change momenta of parts of the system, but cannot change the linear momentum of the entire system
- Do not confuse momentum and energy

9-5 Conservation of Linear Momentum (12 of 12)

Checkpoint 6

An initially stationary device lying on a frictionless floor explodes into two pieces, which then slide across the floor, one of them in the positive *x* direction. (a) What is the sum of the momenta of the two pieces after the explosion? (b) Can the second piece move at an angle to the *x* axis? (c) What is the direction of the momentum of the second piece?

Answer:

(a) zero

(b) no

(c) the negative *x* direction

9 Summary (1 of 5)

Linear Momentum & Newton's 2nd Law

- Linear momentum defined as:

$$\vec{P} = M\vec{v}_{\text{com}}$$ **Equation (9-25)**

- Write Newton's 2nd law:

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt}$$ **Equation (9-27)**

9 Summary (2 of 5)

Collision and Impulse

- Defined as:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt$$ **Equation (9-30)**

- Impulse causes changes in linear momentum

Conservation of Linear Momentum

$$\vec{P} = \text{constant} \quad (\text{closed, isolated system}).$$ **Equation (9-42)**

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