# Fundamentals Physics

**Eleventh Edition**

Halliday

## Chapter 9

Center of Mass and Linear Momentum

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## 9-6 Momentum and Kinetic Energy in Collisions (1 of 8)

**Learning Objectives**

**9.28** Distinguish between elastic collisions, inelastic collisions, and completely inelastic collisions.

**9.29** Identify a one-dimensional collision as one where the objects move along a single axis, both before and after the collision.

**9.30** Apply the conservation of momentum for an isolated one-dimensional collision to relate the initial momenta of the objects to their momenta after the collision.

**9.31** Identify that in an isolated system, the momentum and velocity of the center of mass are not changed even if the objects collide.

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## 9-6 Momentum and Kinetic Energy in Collisions (2 of 8)

- Types of collisions:
- **Elastic collisions**:
  - Total kinetic energy is unchanged (conserved)
  - A useful approximation for common situations
  - In real collisions, some energy is always transferred
- **Inelastic collisions**: some energy is transferred
- **Completely inelastic collisions**:
  - The objects stick together
  - Greatest loss of kinetic energy

 



## 9-6 Momentum and Kinetic Energy in Collisions (3 of 8)

- For one dimension:
- Inelastic collision

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$ **Equation (9-51)**

- Completely inelastic collision, for target at rest:

$$m_1 v_{1i} = (m_1 + m_2) V$$ **Equation (9-52)**

 

## 9-6 Momentum and Kinetic Energy in Collisions (4 of 8)

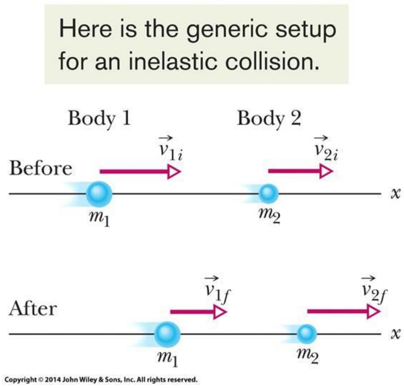


**Figure 9-14**

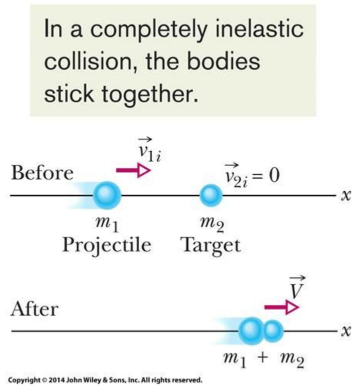


**Figure 9-15**

 



## 9-6 Momentum and Kinetic Energy in Collisions (5 of 8)

$m_1 = 750$ kg 

 $m_2 = 1000$ kg

$v_1 = 20 \frac{\text{m}}{\text{s}}$ (→)

$v_2 = 30 \frac{\text{m}}{\text{s}}$ (←)

Two cars collide and stick together after the collision.
What is the final velocity of the system?

 

# 9-6 Momentum and Kinetic Energy in Collisions (6 of 8)

$m_1 = 750$ kg   $m_2 = 1000$ kg

$v_1 = 20\frac{\text{m}}{\text{s}}$ $\quad$ $v_2 = 30\frac{\text{m}}{\text{s}}$

Using conservation of momentum:

$$P_i = P_f$$

$$m_1 v_{1,i} + m_2 v_{2,i} = (m_1 + m_2)v_f$$

$$v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2} = \frac{(750kg)(20m/s) + (1000kg)(-30m/s)}{(750kg) + (1000kg)}$$

$$v_f = -8.6m/s$$

 



# 9-6 Momentum and Kinetic Energy in Collisions (7 of 8)

- The center of mass velocity remains unchanged:

$$\vec{v}\text{com} = \frac{\vec{P}}{m_1 + m_2} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2}.$$

**Equation (9-56)**

- Figure 9-16 shows freeze frames of a completely inelastic collision, showing center of mass velocity

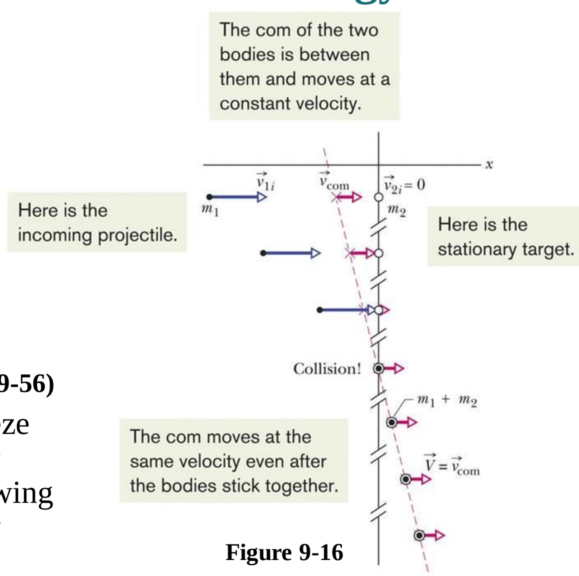


**Figure 9-16**

## 9-6 Momentum and Kinetic Energy in Collisions (8 of 8)

**Checkpoint 7**

Body 1 and body 2 are in a completely inelastic one-dimensional collision. What is their final momentum if their initial momenta are, respectively, (a) 10 kg · m/s and 0; (b) 10 kg · m/s and 4 kg · m/s; (c) 10 kg · m/s and – 4 kg · m/s?

**Answer:**

(a) 10 kg m/s

(b) 14 kg m/s

(c) 6 kg m/s






## 9-7 Elastic Collisions in One Dimension (1 of 12)

**Learning Objectives**

**9.32** For isolated elastic collisions in one dimension, apply the conservation laws for both the total energy and the net momentum of the colliding bodies to relate the initial values to the values after the collision.

**9.33** For a projectile hitting a stationary target, identify the resulting motion for the three general cases: equal masses, target more massive than projectile, projectile more massive than target.

## 9-7 Elastic Collisions in One Dimension (2 of 12)

- Total kinetic energy is conserved in elastic collisions

In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

- For a stationary target, conservation laws give:

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

(linear momentum).

**Equation (9-63)**

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

(kinetic energy).

**Equation (9-64)**

Here is the generic setup for an elastic collision with a stationary target.

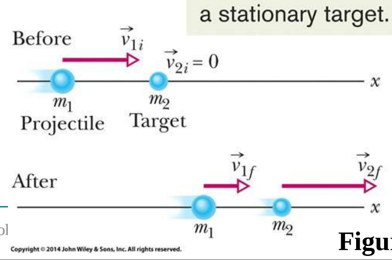


**Figure 9-18**



## 9-7 Elastic Collisions in One Dimension (3 of 12)

- With some algebra we get:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$ **Equation (9-67)**

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$ **Equation (9-68)**

- Results
  - Equal masses: $v_{1f} = 0$, $v_{2f} = v_{1i}$: the first object stops
  - Massive target, $m_2 >> m_1$: the first object just bounces back, speed mostly unchanged
  - Massive projectile:

 

# 9-7 Elastic Collisions in One Dimension (4 of 12)

Here is the generic setup for an elastic collision with a moving target.


Figure 9-19

- For a target that is also moving

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

$$\frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

- After "some" algebra

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$ **Equation (9-75)**

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}.$$ **Equation (9-76)**

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# 9-7 Elastic Collisions in One Dimension (5 of 12)

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$ **Equation (9-75)**

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}.$$ **Equation (9-76)**

- If $m_1 \gg m_2$

$$v_{1f} = v_{2i}$$

$$v_{2f} = 2v_{1i} - v_{2i}.$$

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# 9-7 Elastic Collisions in One Dimension (6 of 12)


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# 9-7 Elastic Collisions in One Dimension (7 of 12)

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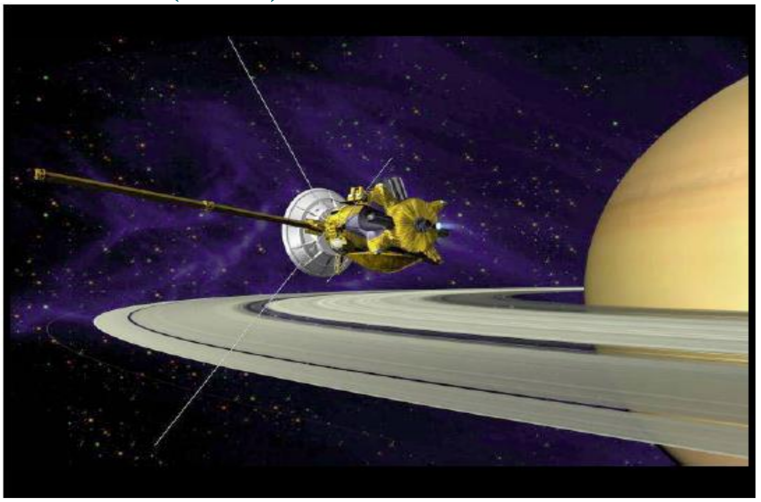
# 9-7 Elastic Collisions in One Dimension (8 of 12)



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# 9-7 Elastic Collisions in One Dimension (9 of 12)

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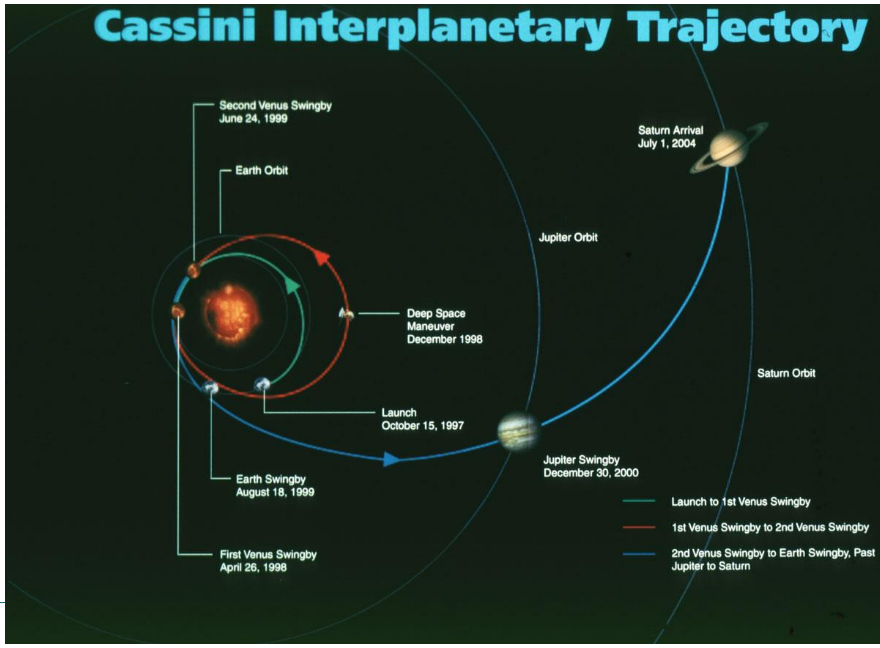


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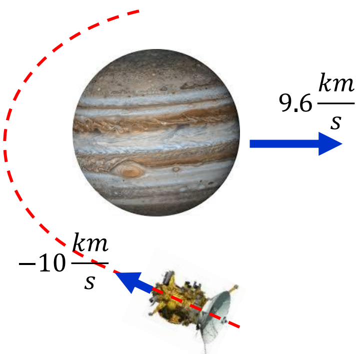
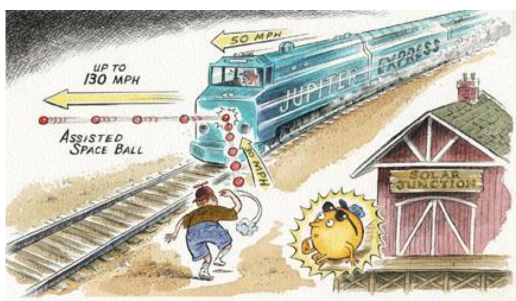
# 9-7 Elastic Collisions in One Dimension (11 of 12)

**Gravity Assist**

$$v_{Cassini,f} = 2v_{jupiter} - v_{Cassini,i}$$
$$= 2(9.6\ km/s) - (-10\ km/s)$$
$$= 29.2\ km/s$$

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## 9-7 Elastic Collisions in One Dimension (12 of 12)

**Checkpoint 8**

What is the final linear momentum of the target in Fig. 9-18 if the initial linear momentum of the projectile is $6\ \text{kg}\cdot\text{m/s}$ and the final linear momentum of the projectile is $(\text{a})\ 2\ \text{kg}\cdot\text{m/s}$ and $(\text{b})\ -2\ \text{kg}\cdot\text{m/s}$? (c) What is the final kinetic energy of the target if the initial and final kinetic energies of the projectile are, respectively, 5 J and 2 J?

**Answer:**

(a) 4 kg m/s

(b) 8 kg m/s

(c) 3 J



## 9-8 Collisions in Two Dimensions (1 of 4)

**Learning Objectives**

**9.34** For an isolated system in which a two-dimensional collision occurs, apply the conservation of momentum along each axis of a coordinate system to relate the momentum components along an axis before the collision to the momentum components along the same axis after the collision.

**9.35** For an isolated system in which a two-dimensional elastic collision occurs, (a) apply the conservation of momentum along each axis to relate the momentum components along an axis before the collision to the momentum components along the same axis after the collision and (b) apply the conservation of total kinetic energy to relate the kinetic energies before and after the collision.

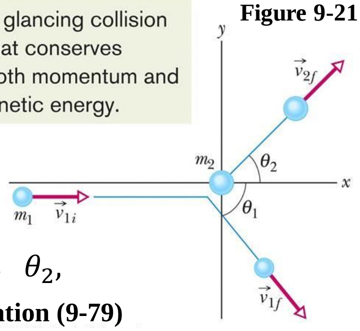
 

# 9-8 Collisions in Two Dimensions (2 of 4)

- Apply the conservation of momentum along each axis
- Apply conservation of energy for elastic collisions

A glancing collision that conserves both momentum and kinetic energy.

**Figure 9-21**




- Along *x*:

$$m_1 v_{1i} = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2,$$

**Equation (9-79)**

- Along *y*:

$$0 = -m_1 v_{1f} \sin \theta_1 + m_2 v_{2f} \sin \theta_2.$$ **Equation (9-80)**

- Energy:

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$ **Equation (9-81)**



# 9-8 Collisions in Two Dimensions (3 of 4)

- Along *x*:

$$m_1 v_{1i} = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2,$$ **Equation (9-79)**

- Along *y*:

$$0 = -m_1 v_{1f} \sin \theta_1 + m_2 v_{2f} \sin \theta_2.$$ **Equation (9-80)**

- Energy:

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$ **Equation (9-81)**

- These 3 equations for a stationary target have 7 unknowns (since $v_{2i} = 0$) : if we know 4 of them we can solve for the remaining ones.

# 9-8 Collisions in Two Dimensions (4 of 4)

**Checkpoint 9**

In Fig. 9-21, suppose that the projectile has an initial momentum of $6\ \text{kg}\cdot\text{m/s}$, a final *x* component of momentum of $4\ \text{kg}\cdot\text{m/s}$, and a final *y* component of momentum of $-3\ \text{kg}\cdot\text{m/s}$. For the target, what then are (a) the final *x* component of momentum and (b) the final *y* component of momentum?

**Answer:**

(a) 2 kg m/s

(b) 3 kg m/s



# 9-9 Systems with Varying Mass: A Rocket (1 of 7)

**Learning Objectives**

**9.36** Apply the first rocket equation to relate the rate at which the rocket loses mass, the speed of the exhaust products relative to the rocket, the mass of the rocket, and the acceleration of the rocket.

**9.37** Apply the second rocket equation to relate the change in the rocket's speed to the relative speed of the exhaust products and the initial and final mass of the rocket.

**9.38** For a moving system undergoing a change in mass at a given rate, relate that rate to the change in momentum.

# 9-9 Systems with Varying Mass: A Rocket (2 of 7)

- Rocket and exhaust products form an isolated system
- After time $dt$, the rocket now has velocity $v + dv$ and mass $M + dM$, where $dM < 0$.
- The velocity $U$ of the exhaust is a velocity with respect to the ground.
- Conserve momentum $P_i = P_f$.
- Rewrite this as:

$$Mv = -dMU + (M + dM)(v + dv),$$

**Equation (9.83)**

- We can simplify using relative speed, defined as:

$$U = v + dv - v_{\text{rel}}.$$

**Equation (9.84)**

The ejection of mass from the rocket's rear increases the rocket's speed.

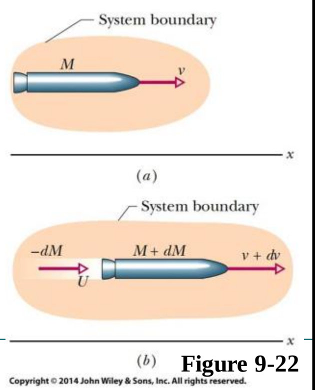



**Figure 9-22**







# 9-9 Systems with Varying Mass: A Rocket (3 of 7)

$$Mv = -dMU + (M + dM)(v + dv),$$

$$U = v + dv - v_{rel}$$

- Substituting yields

$$-dMv_{rel} = Mdv$$

- Or,

$$-\frac{dM}{dt}v_{rel} = M\frac{dv}{dt}$$

- Let $R = -\frac{dM}{dt}$ (positive value) be the mass rate fuel consumption of the rocket, we arrive at the first rocket equation:

$$Rv_{\text{rel}} = Ma$$

**Equation (9-87)**

- The left side of the equation is thrust, $\boldsymbol{T}$.

# 9-9 Systems with Varying Mass: A Rocket (4 of 7)

- From the equation:

$$-dMv_{rel} = Mdv$$

- We rewrite it as

$$dv = -v_{rel}\frac{dM}{M}$$

- Upon integrating we obtain a relation for the increase in the speed of the rocket during the change in mass from $M_i$ to $M_f$, known as the second rocket equation:

$$v_f - v_i = v_{rel}\ln\frac{M_i}{M_f}$$ **Equation (9-88)**



# 9-9 Systems with Varying Mass: A Rocket (5 of 7)

### Rocket engine, thrust, acceleration

In all previous examples in this chapter, the mass of a system is constant (fixed as a certain number). Here is an example of a system (a rocket) that is losing mass. A rocket whose initial mass $M_i$ is 850 kg consumes fuel at the rate $R = 2.3$ kg/s. The speed $v_{rel}$ of the exhaust gases relative to the rocket engine is 2800 m/s. What thrust does the rocket engine provide?

***Calculation:*** Here we find

$$T = Rv_{rel} = (2.3 \text{ kg/s})(2800 \text{ m/s})$$
$$= 6440 \text{ N} \approx 6400 \text{ N}. \qquad \text{(Answer)}$$

# 9-9 Systems with Varying Mass: A Rocket (6 of 7)

**Rocket engine, thrust, acceleration**

What is the initial acceleration of the rocket?

*Calculation:* We find

$$a = \frac{T}{M_i} = \frac{6440\ \text{N}}{850\ \text{kg}} = 7.6\ \text{m/s}^2. \qquad \text{(Answer)}$$






# 9-9 Systems with Varying Mass: A Rocket (7 of 7)

**Rocket engine, thrust, acceleration**

To be launched from Earth's surface, a rocket must have an initial acceleration greater than $g = 9.8\ \text{m/s}^2$. That is, it must be greater than the gravitational acceleration at the surface. Put another way, the thrust $T$ of the rocket engine must exceed the initial gravitational force on the rocket, which here has the magnitude $M_i g$, which gives us

$$(850\ \text{kg})(9.8\ \text{m/s}^2) = 8330\ \text{N}.$$

Because the acceleration or thrust requirement is not met (here $T = 6400$ N), our rocket could not be launched from Earth's surface by itself; it would require another, more powerful, rocket.

# 9 Summary (1 of 5)

**Linear Momentum & Newton's 2nd Law**

- Linear momentum defined as:

$$\vec{P} = M\vec{v}_{\text{com}}$$ **Equation (9-25)**

- Write Newton's 2nd law:

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt}$$ **Equation (9-27)**



# 9 Summary (2 of 5)

**Collision and Impulse**

- Defined as:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt$$ **Equation (9-30)**

- Impulse causes changes in linear momentum

**Conservation of Linear Momentum**

$$\vec{P} = \text{constant} \quad (\text{closed, isolated system}).$$ **Equation (9-42)**

# 9 Summary (3 of 5)

**Inelastic Collision in 1D**

- Momentum conserved along that dimension

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$ **Equation (9-51)**

**Motion of the Center of Mass**

- Unaffected by collisions/internal forces

**Collisions in Two Dimensions**

- Apply conservation of momentum along each axis individually
- Conserve $K$ if elastic



# 9 Summary (4 of 5)

**Elastic Collisions in One Dimension**

- $K$ is also conserved

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$ **Equation (9-67)**

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$ **Equation (9-68)**

# 9 Summary (5 of 5)

### Variable-Mass Systems

$$Rv_{\text{rel}} = Ma \quad \left(\text{first rocket equation}\right).$$ **Equation (9-87)**

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f} \quad \left(\text{second reocket equation}\right)$$ **Equation (9-88)**



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