# Fundamentals Physics

**Eleventh Edition**

Halliday

## Chapter 12

## Equilibrium and Elasticity

1

## 12-1 Equilibrium (1 of 9)

**Learning Objectives**

**12.01** Distinguish between equilibrium and static equilibrium.

**12.02** Specify the four conditions for static equilibrium.

**12.03** Explain center of gravity and how it relates to center of mass.

**12.04** For a given distribution of particles, calculate the coordinates of the center of gravity and the center of mass.

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2

# 12-1 Equilibrium (2 of 9)

- We often want objects to be stable despite forces acting on them
- Consider a book resting on a table, a puck sliding with constant velocity, a rotating ceiling fan, a rolling bicycle wheel with constant velocity
- These objects have the characteristics that:

1. The linear momentum of the center of mass is constant
2. The angular momentum about the center of mass, or any other point, is constant



# 12-1 Equilibrium (3 of 9)

- Such objects are in **equilibrium**

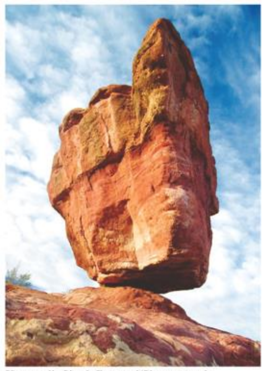
$$\vec{P} = \text{a constant} \quad \text{and} \quad \vec{L} = \text{a constant.}$$ **Equation (12-1)**

- In this chapter we are largely concerned with objects that are not moving at all; $P = L = 0$
- These objects are in **static equilibrium**
- The only one of the examples from the previous page in static equilibrium is the book at rest on the table

## 12-1 Equilibrium (4 of 9)

- As discussed in 8-3, if a body returns to static equilibrium after a slight displacement, it is in stable static equilibrium
- If a small displacement ends equilibrium, it is unstable
- Despite appearances, this rock is in stable static equilibrium, otherwise it would topple at the slightest gust of wind



Kanwarjit Singh Boparai/Shutterstock

**Figure 12-1**



## 12-1 Equilibrium (5 of 9)

- In part (*a*) of the figure, we have unstable equilibrium
- A small force to the right results in (*b*)
- In (*c*) equilibrium is stable, but push the domino so it passes the position shown in (*a*) and it falls
- The block in (*d*) is even more stable

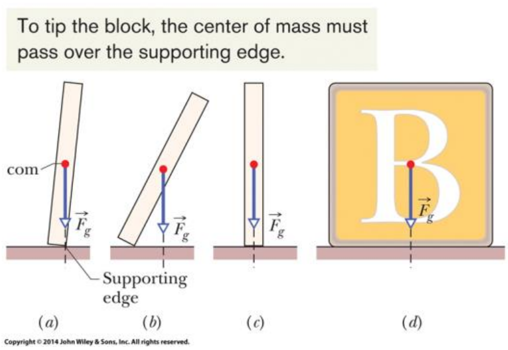



**Figure 12-2**

# 12-1 Equilibrium (6 of 9)

- Requirements for equilibrium are given by Newton's second law, in linear and rotational form

$$\vec{P} = \text{constant} \rightarrow \vec{F}_{\text{net}} = 0 \quad \text{(balance of forces)}. \quad \textbf{Eq. (12-3)}$$

$$\vec{L} = \text{constant} \rightarrow \vec{\tau}_{\text{net}} = 0 \quad \text{(balance of torques)}. \quad \textbf{Eq. (12-5)}$$

- Therefore, we have for equilibrium:

1. The vector sum of all the external forces that act on the body must be zero.
2. The vector sum of all external torques that act on the body, measured about any possible point, must also be zero.



# 12-1 Equilibrium (7 of 9)

- We often simplify matters by considering forces only in the *xy* plane, giving:

$$F_{\text{net},\,x} = 0 \quad \text{(balance of forces)}, \qquad \textbf{Equation (12-7)}$$

$$F_{\text{net},\,y} = 0 \quad \text{(balance of forces)}, \qquad \textbf{Equation (12-8)}$$

$$\tau_{\text{net},\,z} = 0 \quad \text{(balance of torques)}. \qquad \textbf{Equation (12-9)}$$

# 12-1 Equilibrium (8 of 9)

- Note that for static equilibrium we have the additional requirements that:

3. The linear momentum $\vec{P}$ of the body must be zero.
4. The angular momentum of the body $\vec{L}$ must be zero.

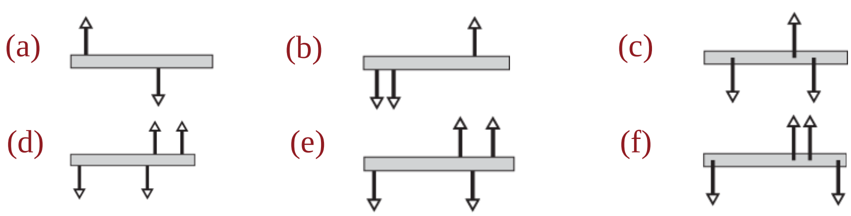
 



# 12-1 Equilibrium (9 of 9)

**Checkpoint 1**

The figure gives six overhead views of a uniform rod on which two or more forces act perpendicularly to the rod. If the magnitudes of the forces are adjusted properly (but kept nonzero), in which situations can the rod be in static equilibrium?




**Answer:**

(c), (e), (f)

# 12-2 Some Examples of Static Equilibrium (1 of 6)

## Learning Objectives

**12.05** Apply the force and torque conditions for static equilibrium.

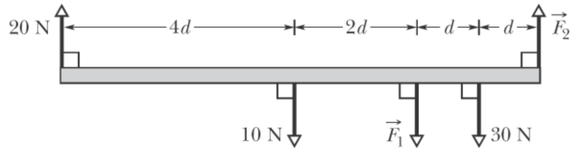
**12.06** Identify that a wise choice about the placement of the origin (about which to calculate torques) can simplify the calculations by eliminating one or more unknown forces from the torque equation.



# 12-2 Some Examples of Static Equilibrium (2 of 6)

## Checkpoint 2




Can you find the magnitudes of unknown forces $\vec{F_1}$ and $\vec{F_2}$ by balancing the forces?

Answer : No

If you wish to find the magnitude of force $\vec{F_2}$ by using a balance torques equation, where should you place a rotation axis to eliminate $\vec{F_1}$ from the equation?

Answer : place the rotation axis at the location where $F_1$ is applied to the beam

The magnitude of $\vec{F_2}$ turns out to be 65 N. What then is the magnitude of $\vec{F_1}$?

Answer : 45 N

## 12-2 Some Examples of Static Equilibrium (3 of 6)

**Example** Balancing a horizontal beam

- $M$ = 2.7 kg, $m$ = 1.8 kg
- Set rotation axis at $x$ = 0
- Sum torques
- $\frac{1}{4}MgL + \frac{1}{2}mgL = F_r L$

  so $F_r = 15$ N
- Balance vertical forces

  $F_l = (M+m)g - F_r = 29\ N$

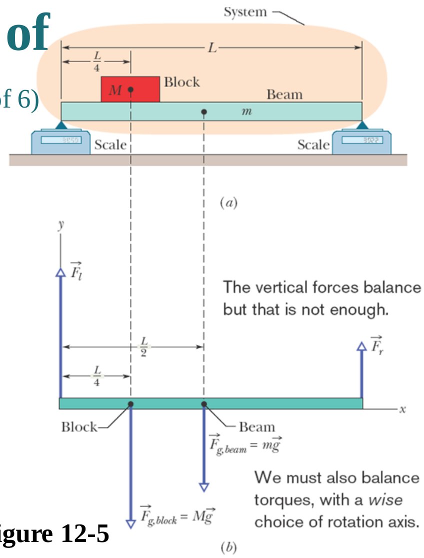



Figure 12-5







## 12-2 Some Examples of Static Equilibrium (4 of 6)

**Example** Balancing a leaning boom:

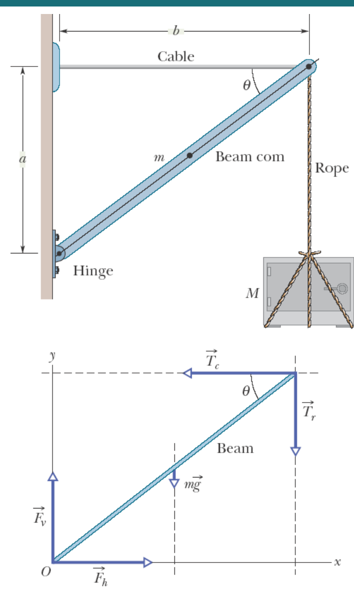
Find the tension in the cable, in the rope and the size of force $F$ by the hinge.

$M$ = 430 kg, $m$ = 85 kg, $a$ = 1.9 $m$, b = 2.5 m

- Set rotation axis at $x$ = 0, $y$ = 0
- Sum torques (using $T_r = Mg$)

$$aT_c - bT_r - \frac{1}{2}bmg = 0$$

$$T_c = 6100 \text{ N}$$

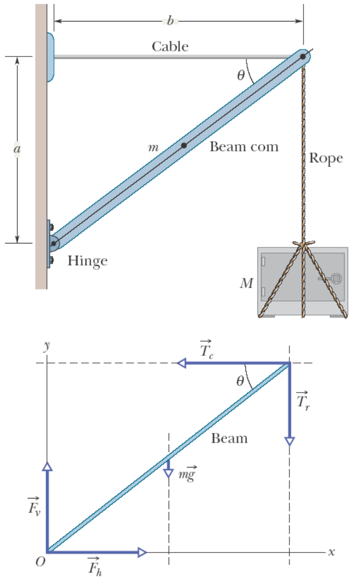


## 12-2 Some Examples of Static Equilibrium (5 of 6)

**Example** Balancing a leaning boom:

Balance forces

$$F_h = T_c = 6100 \text{ N}$$
$$F_v = (m + M)g = 5050 \text{ N}$$
$$F = \sqrt{6100^2 + 5050^2} = 7919 \text{ N}$$





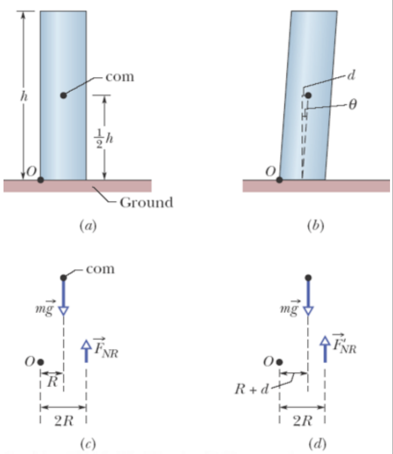

 



## 12-2 Some Examples of Static Equilibrium (6 of 6)

**Example** Leaning Tower of Pisa

- $R$ = 9.8 m, $h$ = 60 m, $\theta$ = 5.5°
- Model: supported by 2 forces, at left and right edges
- No lean: $F_{NR} = \frac{1}{2}mg$
- Lean shifts com by
  $$d = \frac{1}{2}h \tan\theta$$
- New force:
  $$F'_{NR} = \frac{(R + d)mg}{2R}.$$





## 12-3 Elasticity (1 of 12)

**Learning Objectives**

**12.07** Explain what an indeterminate situation is.

**12.08** For tension and compression, apply the equation that relates stress to strain and Young's modulus.

**12.09** Distinguish between yield strength and ultimate strength.

**12.10** For shearing, apply the equation that relates stress to strain and the shear modulus.

**12.11** For hydraulic stress, apply the equation that relates fluid pressure to strain and the bulk modulus.






## 12-3 Elasticity (2 of 12)

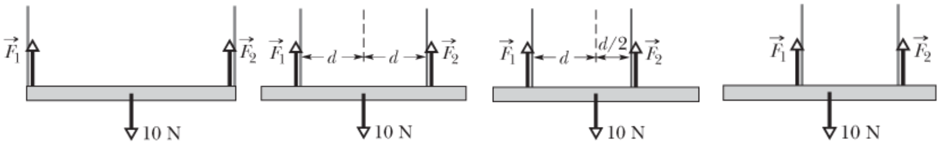
- For problems in the *xy* plane we have 3 independent equations
- Therefore, we can solve for 3 unknowns
- If we have more unknown forces, we cannot solve for them, and the situation is **indeterminate**
- This assumes that bodies are rigid and do not deform (there are no such bodies)
- With some knowledge of elasticity, we can solve more problems

# 12-3 Elasticity (3 of 12)

**Checkpoint 3**

A horizontal uniform bar of weight 10 N is to hang from a ceiling by two wires that exert upward forces $\vec{F}_1$ and $\vec{F}_2$ on the bar.

The figure shows four arrangements for the wires. Which arrangements, if any, are indeterminate (so we cannot solve for numerical values of $\vec{F}_1$ and $\vec{F}_2$ ?




(a) (b) (c) (d)

**Answer:** (d)





# 12-3 Elasticity (4 of 12)

- All rigid bodies are partially **elastic**, meaning we can change their dimensions by applying forces
- A **stress**, deforming force per unit area, produces a **strain**, or unit deformation
- There are 3 main types of stress:
  - Tensile (a), Shearing (b), Hydraulic (c)

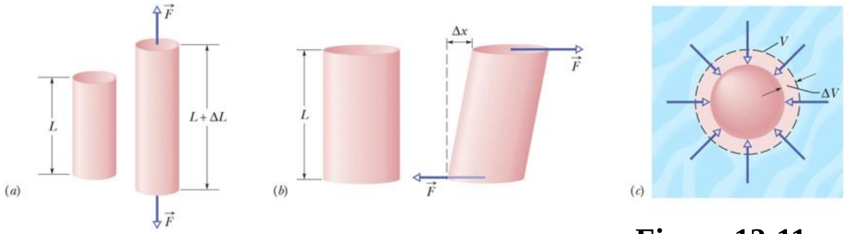





**Figure 12-11**

# 12-3 Elasticity (5 of 12)

- Stress and strain are proportional in the elastic range
- Related by the **modulus of elasticity**:

  stress = modulus × strain.

  **Equation (12-22)**

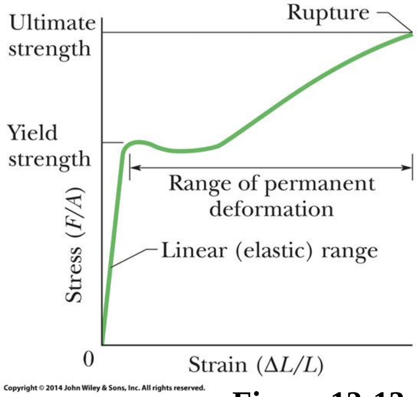





**Figure 12-13**

- As stress increases, eventually a **yield strength** is reached, and the material deforms permanently
- At the **ultimate strength**, the material breaks

 21

21

# 12-3 Elasticity (6 of 12)

- In simple tension/compression, stress is $\frac{F}{A}$
- The strain is the dimensionless quantity $\frac{\Delta L}{L}$
- **Young's modulus**, *E*, used for tension/compression

$$\frac{F}{A} = E\frac{\Delta L}{L}.$$

**Equation (12-23)**

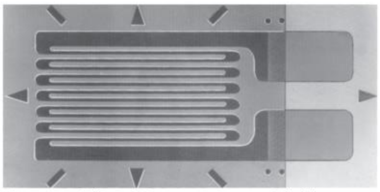
- Note that many materials have very different tensile and compressive strengths, despite the same modulus being used for both
- E.g., concrete: high compressive strength, very low tensile strength

 22

22

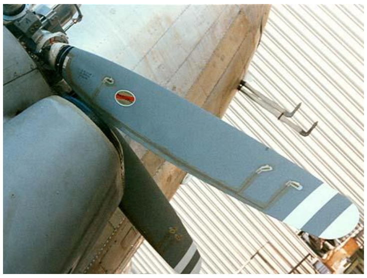
# 12-3 Elasticity (7 of 12)

- Strain can be measured by a strain gage
- Placed on the material, it becomes subject to the same strain
- Strain can be read out as a change in electrical resistance, for strains up to 3%



Courtesy Micro Measurements, a Division of Vishay Precision Group, Raleigh, NC

**Figure 12-14**



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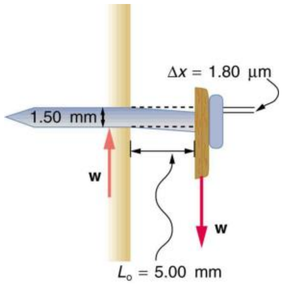
# 12-3 Elasticity (8 of 12)

- **Shear modulus**, $G$, used for shearing

$$\frac{F}{A} = G\frac{\Delta x}{L}.$$ **Equation (12-24)**

- $\Delta x$ is along a different axis than $L$

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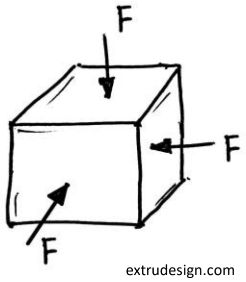
# 12-3 Elasticity (9 of 12)

- **Bulk modulus**, *B*, used for hydraulic compression

$$p = B\frac{\Delta V}{V}. \qquad \textbf{Equation (12-25)}$$

- Relates pressure to volume change

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# 12-3 Elasticity (10 of 12)

- The table shows some elastic properties for common materials, for comparison purposes

**Table 12-1** Some Elastic Properties of Selected Materials of Engineering Interest

| Material | Density $(kg/m^3)$ | Young's Modulus $E$ $(10^9 N/m^2)$ | Ultimate Strength $S_u$ $(10^6 N/m^2)$ | Yield Strength $S_y$ $(10^6 N/m^2)$ |
|---|---|---|---|---|
| Steel[a] | 7860 | 200 | 400 | 250 |
| Aluminum | 2710 | 70 | 110 | 95 |
| Glass | 2190 | 65 | 50[b] | — |
| Concrete[c] | 2320 | 30 | 40[b] | — |
| Wood[d] | 525 | 13 | 50[b] | — |
| Bone | 1900 | 9[b] | 170[b] | — |
| Polystyrene | 1050 | 3 | 48 | — |

[a] Structural steel (A S T M-A36). [b] In compression.

[c] High strength. [d] Douglas fir.



 

# 12-3 Elasticity (11 of 12)

**Example** Balancing a wobbly table

- Three legs of 1.00 m, a fourth longer by 0.50 mm
- Compressed by $M$ = 290 kg so all four legs are compressed but not buckled and the table does not wobble
- Legs are wooden cylinders with area $A$ = 1.0 sq cm
- $E = 1.3 \times 10^{10}\,\mathrm{N/m^2}$
- The 3 shorter legs must compress the same amount, the longer leg compresses more
- Write length comparison, use the stress-strain equation, and approximate all legs to be length $L$

$$\frac{F_4 L}{AE} = \frac{F_3 L}{AE} + d.$$ **Equation (12-27)**

 



# 12-3 Elasticity (12 of 12)

**Example** Balancing a wobbly table (continued)

- Get a second equation by balancing forces

$$3F_3 + F_4 - Mg = 0,$$ **Equation (12-28)**

- Solve the simultaneous equations to find
  - $F_3$ = 550 N
  - $F_4$ = 1200 N
- Each short leg is compressed by 0.42 mm, and the long leg is compressed by 0.92 mm

 

# Summary (1 of 3)

## Static Equilibrium

$$\vec{F}_{\text{net}} = 0 \quad (\text{balance of forces}).$$ **Equation (12-3)**

$$\vec{\tau}_{\text{net}} = 0 \quad (\text{balance of torques}).$$ **Equation (12-5)**







# Summary (2 of 3)

## Elastic Moduli

- Three elastic moduli
- Strain: fractional length change
- Stress: force per unit area

$$\text{stress} = \text{modules} \times \text{strain}.$$ **Equation (12-22)**

## Tension and Compression

- $E$ is Young's modulus

$$\frac{F}{A} = E\frac{\Delta L}{L}.$$ **Equation (12-23)**

# Summary (3 of 3)

### Shearing

- $G$ is the shear modulus

$$\frac{F}{A} = G\frac{\Delta x}{L}.$$ **Equation (12-24)**

### Hydraulic Stress

- $B$ is the bulk modulus

$$p = B\frac{\Delta V}{V}.$$ **Equation (12-25)**





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