Fundamentals Physics

Eleventh Edition

Halliday

Chapter 16

Waves - I

16-1 Transverse Waves (1 of 11)

Learning Objectives

16.01 Identify the three main types of waves.

16.02 Distinguish between transverse waves and longitudinal waves.

16.03 Given a displacement function for a traverse wave, determine amplitude y_m, angular wave number k, angular frequency ω, phase constant ϕ, and direction of travel, and calculate the phase $kx \pm t + \phi$ and the displacement at any given time and position.

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16-1 Transverse Waves (2 of 11)

16.04 Given a displacement function for a traverse wave, calculate the time between two given displacements.

16.05 Sketch a graph of a transverse wave as a function of position, identifying amplitude y_m wavelength.

λ, where the slope is greatest, where it is zero, and where the string elements have positive velocity, negative velocity, and zero velocity.

16.06 Given a graph of displacement versus time for a transverse wave, determine amplitude y_m and period T.

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16-1 Transverse Waves (3 of 11)

16.07 Describe the effect on a transverse wave of changing phase constant ϕ.

16.08 Apply the relation between the wave speed v, the distance traveled by the wave, and the time required for that travel.

16.09 Apply the relationships between wave speed v, angular frequency ω, angular wave number k, wavelength λ, period T, and frequency f.

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16-1 Transverse Waves (4 of 11)

16.10 Describe the motion of a string element as a transverse wave moves through its location, and identify when its transverse speed is zero and when it is maximum.

16.11 Calculate the transverse velocity $u(t)$ of a string element as a transverse wave moves through its location.

16.12 Calculate the transverse acceleration $a(t)$ of a string element as a transverse wave moves through its location.

16.13 Given a graph of displacement, transverse velocity, or transverse acceleration, determine the phase constant ϕ.

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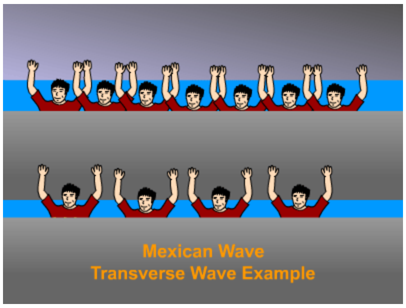
16-1 Transverse Waves (3 of 11)

Waves and Particles

Particle: concentration of matter, can transmit energy.

Wave: broad distribution of energy, filling the space through which it travels → disturbance that travels (not the medium).



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16-1 Transverse Waves (5 of 11)

Types of Waves

1. **Mechanical Waves:** They are governed by Newton's laws, and they can exist only within a material medium, such as water, air, and rock. Examples: water waves, sound waves, and seismic waves.
2. **Electromagnetic waves:** These waves require no material medium to exist. Light waves from stars, for example, travel through the vacuum of space to reach us. All electromagnetic waves travel through a vacuum at the same speed c = 299 792 458 m/s.
3. **Matter waves:** These waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules. Because we commonly think of these particles as constituting matter, such waves are called matter waves.

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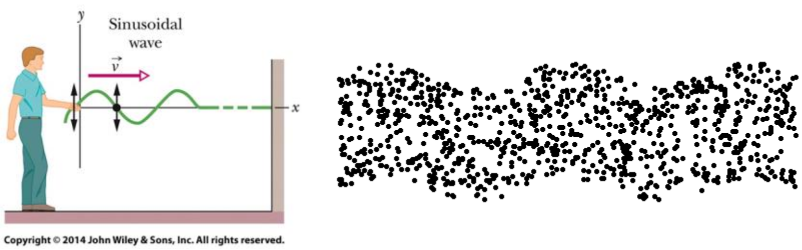
16-1 Transverse Waves (6 of 11)

Transverse and Longitudinal Waves

A sinusoidal wave is sent along the string (Figure (a)). A typical string element moves up and down continuously as the wave passes. This is **transverse wave.**



(a) Transverse Wave

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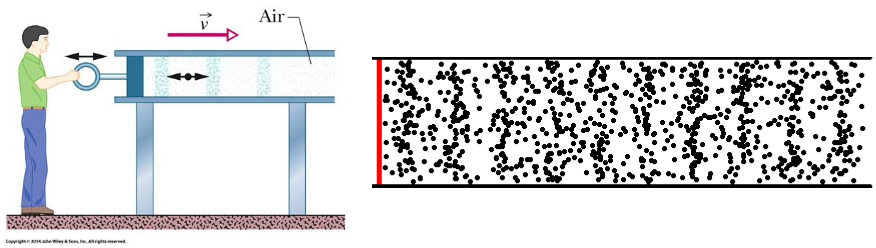
16-1 Transverse Waves (7 of 11)

A sound wave is set up in an air- filled pipe by moving a piston back and forth (Figure (b)). Because the oscillations of an element of the air (represented by the dot) are parallel to the direction in which the wave travels, the wave is a **longitudinal wave.**



(b) Longitudinal Wave

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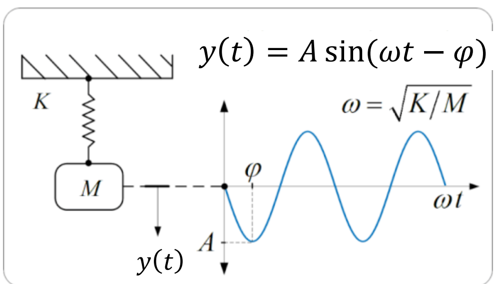
16-1 Transverse Waves (8 of 11)

Generating and Representing Waves

- Because a wave is a traveling disturbance, this means that the mathematical function representing a wave should be a function of space and time variable, x and t.



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- (Periodic) waves mathematically are represented in terms of harmonic functions (sine or cosine).
- Non-periodic wave can be represented as a summation of various harmonic waves.

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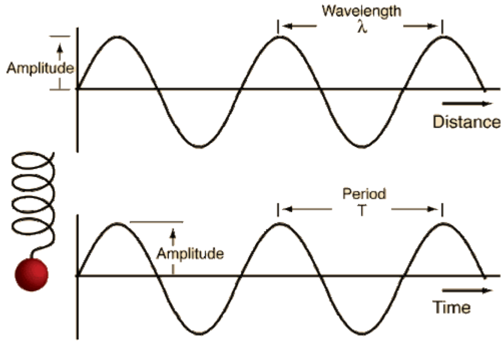
16-1 Transverse Waves (8 of 11)

Harmonic Waves Characteristics

- Wavelength: The distance λ between identical points on the wave.
- Amplitude: The maximum displacement *A* of a point on the wave.
- Period: The time *T* for a point on the wave to undergo one complete oscillation.



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16-1 Transverse Waves (8 of 11)

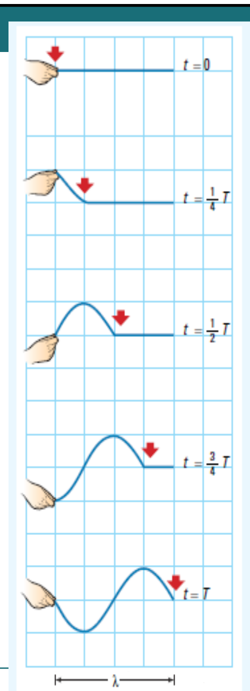
Harmonic Waves Characteristics

- Speed: The wave moves one wavelength λ in one period T so its speed is $v = \frac{\lambda}{T}$.

$$\lambda = vT$$

$$v = \frac{\lambda}{T} = \lambda f$$

- Frequency, f: is number of periods per second (Hertz, Hz)



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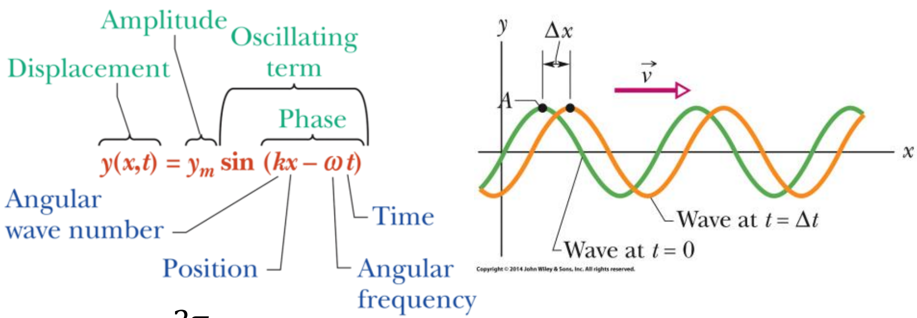
16-1 Transverse Waves (8 of 11)

Harmonic Travelling Waves

$$y(x,t) = y_m \sin(kx - \omega t)$$



$$k = \frac{2\pi}{\lambda} \quad \text{(angular wave number, spatial frequency).}$$

$$\omega = \frac{2\pi}{T} \quad \text{(angular frequency).} \qquad f = \frac{1}{T} = \frac{\omega}{2\pi} \quad \text{(frequency).}$$

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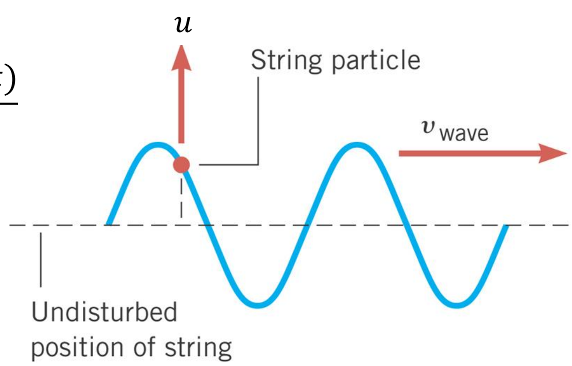
16-1 Transverse Waves (8 of 11)

Oscillation Speed

- The propagation speed of a wave on a string is not the same as the speed at which a particle on the string moves.

$$u = \frac{\partial y(x,t)}{\partial t}$$



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16-1 Transverse Waves (8 of 11)

Example

(a) Write an expression describing a sinusoidal transverse wave traveling on a cord in the $+y$ direction with an wave number of 60 cm^{-1}, a period of 0.20 s, and an amplitude of 3.0 mm. Take the transverse direction to be the z direction.

(b) What is the maximum transverse speed of a point on the cord?

(a) $k = 60\ \text{cm}^{-1}, T = 0.2\ \text{s}, z_m = 3.0\ \text{mm}$

$z(y,t) = z_m sin(ky - \omega t)$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2}\ \text{s} = 10\pi\ \text{s}^{-1}$$

$$z(y,t) = (3.0\ \text{mm}) \sin[(60\ \text{cm}^{-1})y - (10\pi\ \text{s}^{-1})t]$$

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16-1 Transverse Waves (8 of 11)

Example

(a) Write an expression describing a sinusoidal transverse wave traveling on a cord in the $+y$ direction with an wave number of 60 cm^{-1}, a period of 0.20 s, and an amplitude of 3.0 mm. Take the transverse direction to be the z direction.

(b) What is the maximum transverse speed of a point on the cord?

(b) Speed

$$u = \frac{\partial z(y,t)}{\partial t} = -\omega z_m \cos(ky - \omega t)$$

$$u_{max} = \omega z_m = 94\ \frac{\text{mm}}{\text{s}}$$

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16-2 Wave Speed on a Stretched String (1 of 2)

Learning Objectives

16.14 Calculate the linear density μ of a uniform string in terms of the total mass and total length.

16.15 Apply the relationship between wave speed v, tension τ, and linear density μ.

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16-2 Wave Speed on a Stretched String (2 of 2)

The speed of a wave on a stretched string is set by properties of the string (i.e., linear density) and the force stretching the string, not properties of the wave such as frequency or amplitude.

$$v = \sqrt{\frac{\tau}{\mu}},$$

$$\mu = \frac{m}{l}$$

l: length of string, τ: tension on the string

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16-3 Energy and Power of a Wave Traveling along a String (1 of 3)

Learning Objective

16.16 Calculate the average rate at which energy is transported by a transverse wave.

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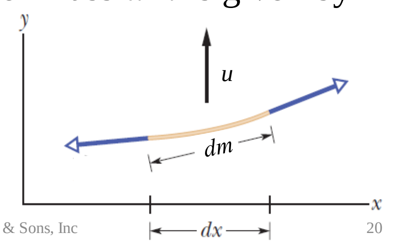
16-3 Energy and Power of a Wave Traveling along a String (2 of 3)

- When we set up a wave on a stretched string, we provide energy for the motion of the string. As the wave moves away from us, it transports that energy as both kinetic energy and elastic potential energy.
- **The Rate of Energy Transmission** The kinetic energy dK associated with a string element of mass dm is given by

$$dK = \frac{1}{2} dm\ u^2,$$

where u is the transverse speed of the oscillating string element



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16-3 Energy and Power of a Wave Traveling along a String (2 of 3)

$$dK = \frac{1}{2} dm\, u^2 \qquad dm = \mu dx \qquad y(x,t) = y_m \sin(kx - \omega t)$$

$$u = \frac{\partial y(x,t)}{\partial t} = -\omega y_m \cos(kx - \omega t)$$

$$dK = \frac{1}{2} dm u^2 = \frac{1}{2}(\mu dx)(-\omega y_m)^2 \cos^2(kx - \omega t)$$

$$\frac{dK}{dt} = \frac{1}{2}\left(\mu \frac{dx}{dt}\right)(-\omega y_m)^2 \cos^2(kx - \omega t)$$

$$= \frac{1}{2}\mu v \omega^2 y_m^2 \cos^2(kx - \omega t), \quad v = \frac{dx}{dt}$$

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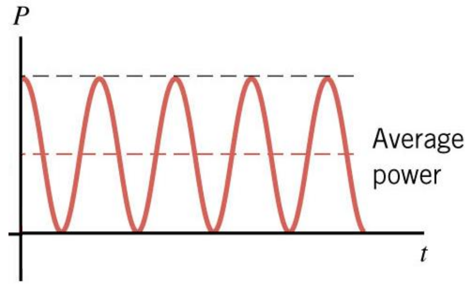
16-3 Energy and Power of a Wave Traveling along a String (2 of 3)

$$\left(\frac{dK}{dt}\right)_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2 [\cos^2(kx - \omega t)]_{avg} = \frac{1}{4}\mu v \omega^2 y_m^2$$



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16-3 Energy and Power of a Wave Traveling along a String (3 of 3)

- The **average power** of, or average rate at which energy is transmitted by, a sinusoidal wave on a stretched string is given by

$$P_{avg} = 2\left(\frac{dK}{dt}\right)_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2$$

The factors μ and v in this equation depend on the material and tension of the string. The factors ω and y_m depend on the process that generates the wave.

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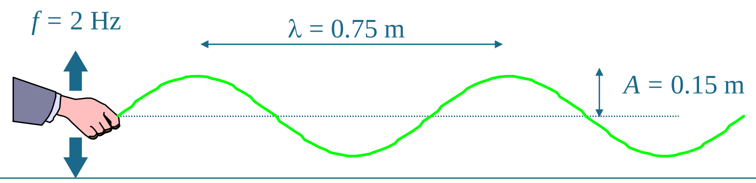
16-3 Energy and Power of a Wave Traveling along a String (3 of 3)

Example

A rope with a mass of $\mu = 0.2$ kg/m lays on a frictionless floor. You grab one end and shake it from side to side *twice per second* with an amplitude of 0.15 m. You notice that the distance between adjacent crests on the wave you make is 0.75 m.

a. What is the average power you are providing the rope?

b. What is the tension in the rope?



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16-3 Energy and Power of a Wave Traveling along a String (3 of 3)

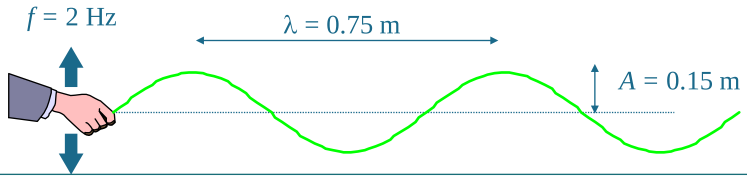
a. What is the average power you are providing the rope?

We know y_m, μ and $\omega = 2\pi f$. We need to find v!

$$v = \lambda f = (.75\text{ m})(2\text{ s}^{-1}) = 1.5\text{ m/s}\ .$$

Hence, the average power is

$$P_{ave} = \tfrac{1}{2}\mu v\omega^2 y_m^2 = \tfrac{1}{2}(0.2)(1.5)(4\pi)^2(0.15)^2$$
$$= 0.533\text{ W}$$



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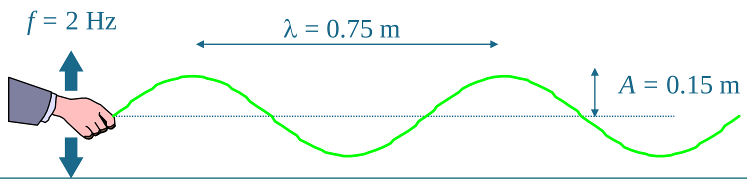
16-3 Energy and Power of a Wave Traveling along a String (3 of 3)

b. What is the tension in the rope?

$$v = \sqrt{\frac{\tau}{\mu}} \quad \rightarrow \quad \tau = \mu v^2 = (0.2)(1.5)^2 = 0.45\text{ N}$$



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16-4 The Wave Equation (1 of 4)

Learning Objective

16.17 For the equation giving a string-element displacement as a function of position x and time t, apply the relationship between the second derivative with respect to x and the second derivative with respect to t.

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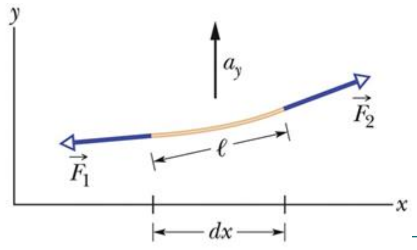
16-4 The Wave Equation (2 of 4)

By applying Newton's second law to the element's motion, we can derive a general differential equation, called the wave equation, that governs the travel of waves of any type.

(a) A string element as a sinusoidal transverse wave travels on a stretched string. Forces $\vec{F_1}$ and $\vec{F_2}$ act at the left and right ends, producing acceleration with a vertical component a_y.



(a)

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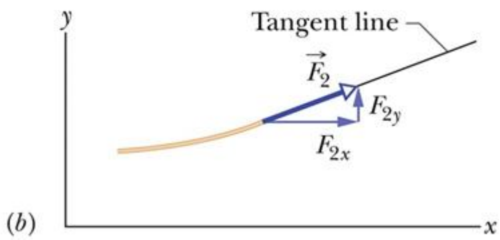
16-4 The Wave Equation (2 of 4)

(b) The force at the element's right end is directed along a tangent to the element's right side.



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16-4 The Wave Equation (2 of 4)

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \text{(wave equation).}$$

This is the general differential equation that governs the travel of waves of all types. Here the waves travel along an *x* axis and oscillate parallel to the *y* axis, and they move with speed *v*, in either the positive *x* direction or the negative *x* direction.

16-5 Interference of Waves (1 of 6)

Learning Objective

16.18 Apply the principle of superposition to show that two overlapping waves add algebraically to give a resultant (or net) wave.

16.19 For two transverse waves with the same amplitude and wavelength and that travel together, find the displacement equation for the resultant wave and calculate the amplitude in terms of the individual wave amplitude and the phase difference.

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16-5 Interference of Waves (2 of 6)

16.20 Describe how the phase difference between two transverse waves (with the same amplitude and wavelength) can result in fully constructive interference, fully destructive interference, and intermediate interference.

16.21 With the phase difference between two interfering waves expressed in terms of wavelengths, quickly determine the type of interference the waves have.

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16-5 Interference of Waves (3 of 6)

Principle of Superposition of waves

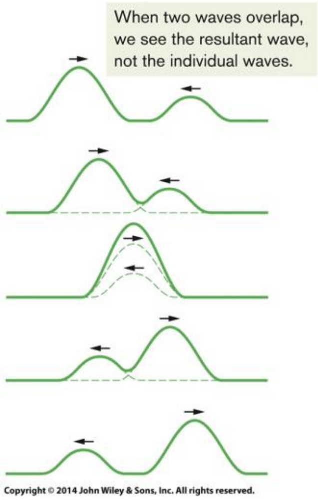
Let $y_1(x, t)$ and $y_2(x, t)$ be the displacements that the string would experience if each wave traveled alone. The displacement of the string when the waves overlap is then the algebraic sum

$$y(x,t) = y_1(x,t) + y_2(x,t).$$

This summation of displacements along the string means that

Overlapping waves algebraically add to produce a **resultant wave** (or **net wave**).

Overlapping waves do not in any way alter the travel of each other.



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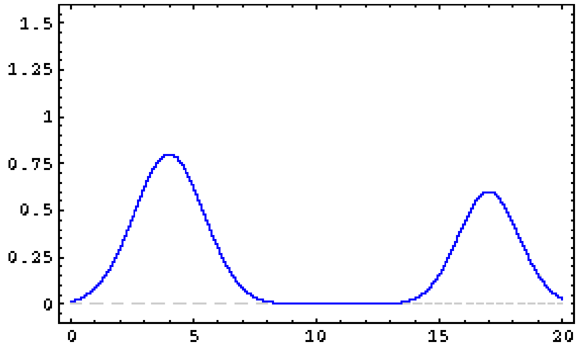
16-5 Interference of Waves (3 of 6)

$$y(x,t) = y_1(x,t) + y_2(x,t)$$



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$$\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$$

16-5 Interference of Waves (4 of 6)

Interference of Wave

Two waves, same amplitude, wavelength, speed, but different phase

$$y_1(x,t) = y_m \sin(kx - \omega t) \qquad y_2(x,t) = y_m \sin(kx - \omega t + \varphi)$$

$$y(x,t) = y_1 + y_2 = \left[2y_m \cos\frac{1}{2}\varphi\right] \sin\left(kx - \omega t + \frac{1}{2}\varphi\right)$$

Displacement

$$y'(x,t) = [2y_m \cos\tfrac{1}{2}\phi]\,\sin(kx - \omega t + \tfrac{1}{2}\phi)$$

Magnitude gives amplitude — Oscillating term

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16-5 Interference of Waves (5 of 6)

Constructive and Destructive Interference

$$y(x,t) = y_1 + y_2 = \left[2y_m \cos\frac{1}{2}\varphi\right] \sin\left(kx - \omega t + \frac{1}{2}\varphi\right)$$

Constructive: $\varphi = n(2\pi)$ — ***Amplitude* = 2*y*$_m$**

Destructive: $\varphi = \left(n + \frac{1}{2}\right)(2\pi)$ — ***Amplitude*=0**

$n = 0,1,2, \ldots$

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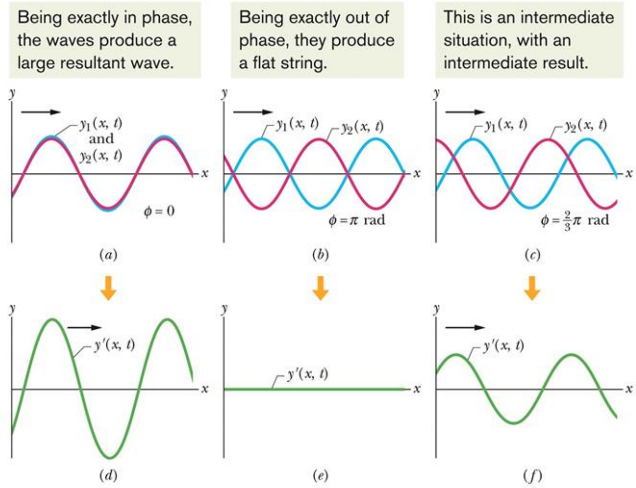
16-5 Interference of Waves (5 of 6)

Constructive and Destructive Interference



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16-5 Interference of Waves (5 of 6)

Example

Two identical traveling waves, moving in the same direction, are out of phase by $\pi/2$ rad. What is the amplitude of the resultant wave in terms of the common amplitude y_m of the two combining waves?

$$y_1(x,t) = y_m \sin(kx - \omega t) \qquad y_2(x,t) = y_m \sin(kx - \omega t + \varphi)$$

$$y(x,t) = y_1 + y_2 = \left[2y_m \cos\frac{1}{2}\varphi\right] \sin\left(kx - \omega t + \frac{1}{2}\varphi\right)$$

For $\varphi = \frac{\pi}{2}$

$$A = 2y_m \cos\frac{1}{2}\varphi = 2y_m \cos\frac{\pi}{4} = 1.4y_m$$

16-6 Phasors (1 of 3)

Learning Objective

16.22 Using sketches, explain how a phasor can represent the oscillations of a string element as a wave travels through its location.

16.23 Sketch a phasor diagram for two overlapping waves traveling together on a string, indicating their amplitudes and phase difference on the sketch.

16.24 By using phasors, find the resultant wave of two transverse waves traveling together along a string, calculating the amplitude and phase and writing out the displacement equation, and then displaying all three phasors in a phasor diagram that shows the amplitudes, the leading or lagging, and the relative phases.

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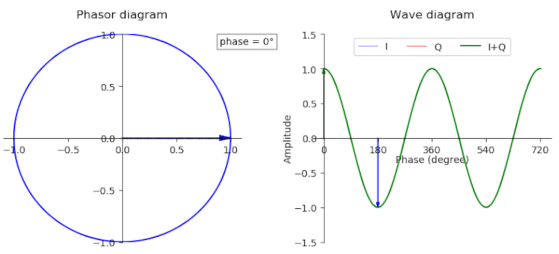
16-6 Phasors (2 of 3)

A phasor is a vector that rotates around its tail, which is pivoted at the origin of a coordinate system. The magnitude of the vector is equal to the amplitude y_m of the wave that it represents.



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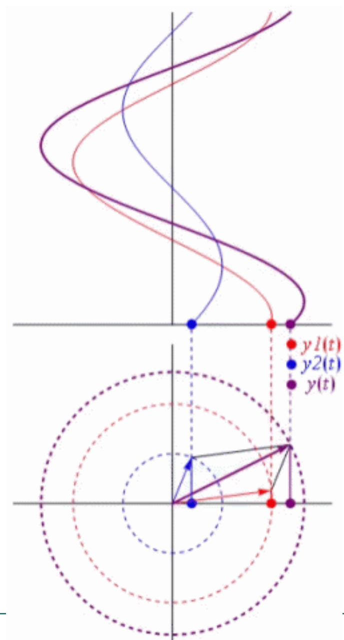

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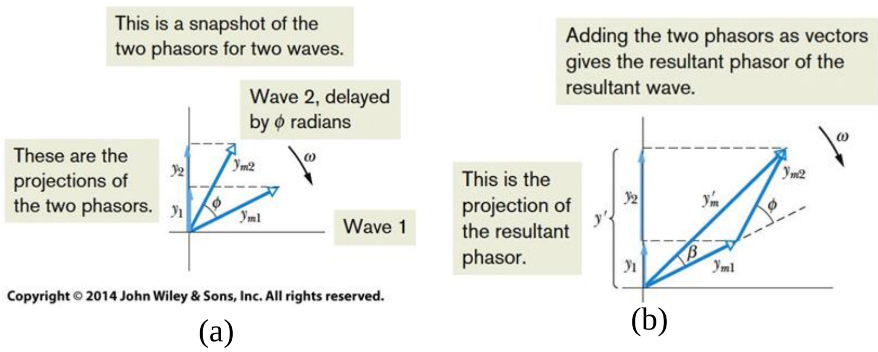
16-6 Phasors (2 of 3)



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16-7 Standing Waves and Resonance
(1 of 4)

Learning Objective

16.25 For two overlapping waves (same amplitude and wavelength) that are traveling in opposite directions, sketch snapshots of the resultant wave, indicating nodes and antinodes.

16.26 For two overlapping waves (same amplitude and wavelength) that are traveling in opposite directions, find the displacement equation for the resultant wave and calculate the amplitude in terms of the individual wave amplitude.

16.27 Describe the SHM of a string element at an antinode of a standing wave.

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16-7 Standing Waves and Resonance
(2 of 4)

16.28 For a string element at an antinode of a standing wave, write equations for the displacement, transverse velocity, and transverse acceleration as functions of time.

16.29 Distinguish between "hard" and "soft" reflections of string waves at a boundary.

16.30 Describe resonance on a string tied taut between two supports, and sketch the first several standing wave patterns, indicating nodes and antinodes.

16.31 In terms of string length, determine the wavelengths required for the first several harmonics on a string under tension.

16.32 For any given harmonic, apply the relationship between frequency, wave speed, and string length.

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$$\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$$

16-7 Standing Waves and Resonance
(3 of 4)

Standing Waves

The interference of two identical sinusoidal waves moving in opposite directions produces standing waves. For a string with fixed ends, the standing wave is given by

$$y_1(x,t) = y_m \sin(kx - \omega t) \qquad y_2(x,t) = y_m \sin(kx + \omega t)$$

$$y(x,t) = y_1 + y_2 = [2y_m \sin kx] \cos \omega t$$

Amplitude depends on position

The wave does not travel

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16-7 Standing Waves and Resonance
(3 of 4)

$$\sin(n\pi) = 0 \quad \sin\left[\left(n + \frac{1}{2}\right)\pi\right] = 1$$

$$y(x,t) = [2y_m \sin kx] \cos \omega t$$

NODES: points of zero amplitude

$$kx = n\pi, \quad \text{or} \quad x = \frac{n\lambda}{2} \quad n = 0,1,2,\ldots \qquad k = \frac{2\pi}{\lambda}$$

ANTINODES: points of maximum ($2y_m$) amplitude

$$kx = \left(n + \frac{1}{2}\right)\pi, \text{ or } x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2} \quad n = 0,1,2,\ldots$$

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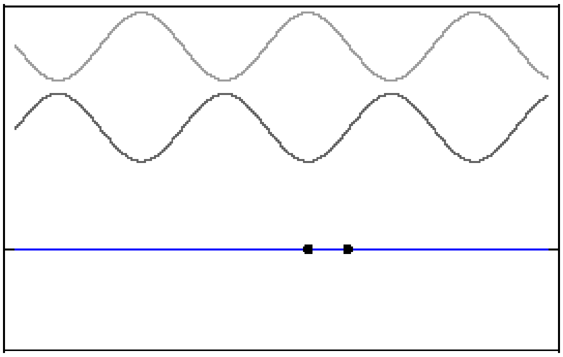
16-7 Standing Waves and Resonance
(3 of 4)

$$y(x,t) = [2y_m \sin kx] \cos \omega t$$



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16-7 Standing Waves and Resonance
(3 of 4)

Harmonics for the case of both ends fixed

$$y(x,t) = [2y_m \sin kx] \cos \omega t$$

$y(x = 0) = 0$ $\qquad$ $y(x = L) = 0$

$\sin(kL) = 0 \longrightarrow k = \frac{n\pi}{L}, \quad n = 1,2,3,\ldots.$

k can only take these values

OR $\qquad \lambda = \frac{2L}{n} \qquad k = \frac{2\pi}{\lambda}$

OR $\qquad f = \frac{v}{\lambda} = \frac{nv}{2L}$ $\qquad$ where $\qquad v = \sqrt{\frac{\tau}{\mu}}$

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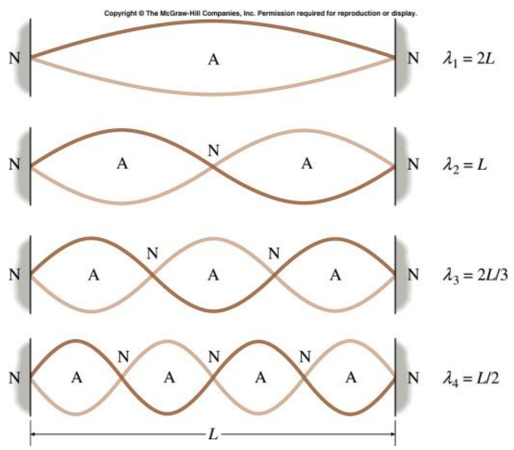
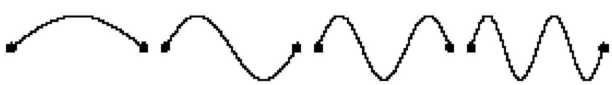
16-7 Standing Waves and Resonance
(3 of 4)

Harmonics

- Fundamental $n = 1$
- $\lambda_n = 2L/n$
- $f_n = nv/(2L)$





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2D Standing Wave

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Summary (1 of 4)

Waves

- Transverse Waves
- Longitudinal Waves

Wave Speed

- $\dfrac{\text{Angular velocity}}{\text{Angular wave number}}$

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f.$$

Equation (16-13)

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Summary (2 of 4)

Sinusoidal Waves

- Wave moving in positive direction (vector)

$$y(x,t) = y_m \sin(kx - \omega t)$$ **Equation (16-2)**

Traveling Waves

- A functional form for traveling waves

$$y(x,t) = h(kx \pm \omega t)$$ **Equation (16-17)**

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Summary (3 of 4)

Powers

- Average Power is given by

$$P_{\text{avg}} = \tfrac{1}{2}\mu v \omega^2 y^2{}_m$$ **Equation (16-33)**

Standing Waves

- The interference of two identical sinusoidal waves moving in opposite directions produces standing waves.

$$y'(x,t) = [2y_m \sin kx]\cos \omega t.$$ **Equation (16-60)**

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Summary (4 of 4)

Interference of Waves

- Two sinusoidal waves on the same string exhibit interference

$$y'(x,t) = \left[2y_m \cos\tfrac{1}{2}\phi\right]\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right).$$ **Equation (16-51)**

Resonance

- For a stretched string of length *L* with fixed ends, the resonant frequencies are

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \quad \text{for } n = 1, 2, 3, \ldots$$ **Equation (16-66)**

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