Fundamentals Physics

Eleventh Edition

Halliday

Chapter 4

Motion in Two and Three Dimensions

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4-1 Position and Displacement (1 of 4)

Learning Objectives

4.01 Draw two-dimensional and three-dimensional position vectors for a particle, indicating the components along the axes of a coordinate system.

4.02 On a coordinate system, determine the direction and magnitude of a particle's position vector from its components, and vice versa.

4.03 Apply the relationship between a particle's displacement vector and its initial and final position vectors.

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4-1 Position and Displacement (2 of 4)

- A **position vector** locates a particle in space
  - Extends from a reference point (origin) to the particle

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, \qquad \textbf{Equation (4-1)}$$

Example

Position vector (−3m, 2m, 5m)

$$\vec{r} = (-3\text{ m})\hat{i} + (2\text{ m})\hat{j} + (5\text{ m})\hat{k}$$

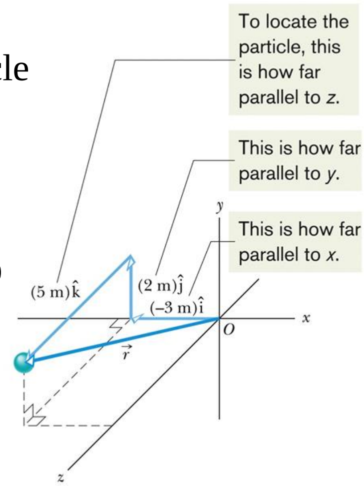


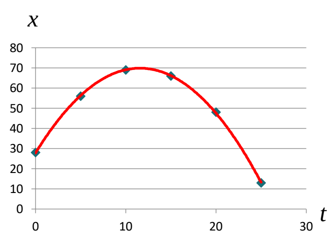
Figure 4-1

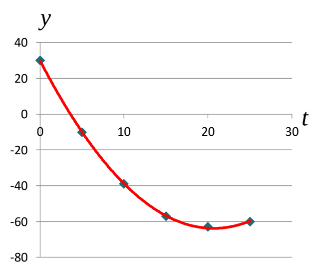
4-1 Position and Displacement (3 of 4)

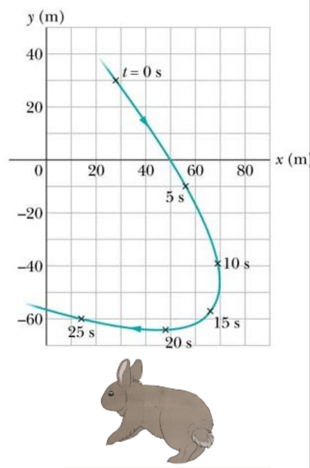
Position of particle is specified by

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

which is a vector depending on time.
Can be separated into motion for each separate direction.







$x = -0.31t^2 + 7.2t + 28$
$y = 0.22t^2 - 9.1t + 30.$

4-1 Position and Displacement (4 of 4)

- Change in position vector is a **displacement**

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1. \qquad \textbf{Equation (4-2)}$$

- We can rewrite this as:

$$\Delta\vec{r} = (x_2 - x_1)\hat{\mathrm{i}} + (y_2 - y_1)\hat{\mathrm{j}} + (z_2 - z_1)\mathrm{k}, \qquad \textbf{Equation (4-3)}$$

- Or express it in terms of changes in each coordinate:

$$\Delta\vec{r} = \Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\mathrm{k}. \qquad \textbf{Equation (4-4)}$$

4-2 Average Velocity and Instantaneous Velocity (1 of 10)

Learning Objectives

4.04 Identify that velocity is a vector quantity and thus has both magnitude and direction and also has components.

4.05 Draw two-dimensional and three-dimensional velocity vectors for a particle, indicating the components along the axes of the coordinate system.

4.06 In magnitude-angle and unit-vector notations, relate a particle's initial and final position vectors, the time interval between those positions, and the particle's average velocity vector.

4.07 Given a particle's position vector as a function of time, determine its (instantaneous) velocity vector.

4-2 Average Velocity and Instantaneous Velocity (2 of 10)

- **Average velocity** is
  - A displacement divided by its time interval

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}.$$ **Equation (4-8)**

- We can write this in component form:

$$\vec{v}_{avg} = \frac{\Delta x \hat{\mathrm{i}} + \Delta y \hat{\mathrm{j}} + \Delta z \mathrm{k}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\mathrm{i}} + \frac{\Delta y}{\Delta t}\hat{\mathrm{j}} + \frac{\Delta z}{\Delta t}\mathrm{k}.$$ **Equation (4-9)**

4-2 Average Velocity and Instantaneous Velocity (3 of 10)

Example

- A particle moves through displacement

$$(12\ m)\hat{\imath} + (3.0\ m)\hat{k}$$

in 2.0 s:

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{(12\ \mathrm{m})\hat{\mathrm{i}} + (3.0\ \mathrm{m})\mathrm{k}}{2.0\mathrm{s}} = (6.0\mathrm{m/s})\hat{\mathrm{i}} + (1.5\mathrm{m/s})\mathrm{k}.$$

4-2 Average Velocity and Instantaneous Velocity (4 of 10)

- **Instantaneous velocity** is
  - The velocity of a particle at a single point in time
  - The limit of avg. velocity as the time interval shrinks to 0

$$\vec{v} = \frac{d\vec{r}}{dt}.$$ **Equation (4-10)**

4-2 Average Velocity and Instantaneous Velocity (5 of 10)

- Visualize displacement and instantaneous velocity:

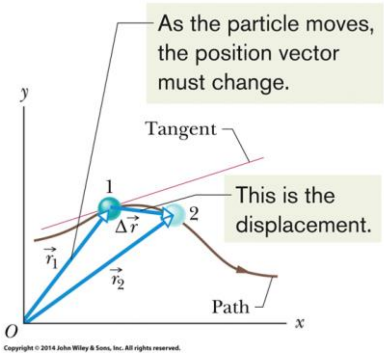


Figure 4-3

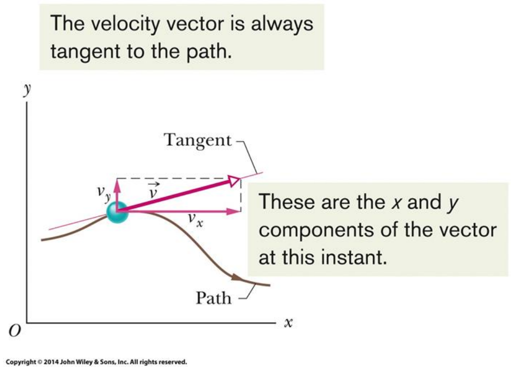


Figure 4-4

4-2 Average Velocity and Instantaneous Velocity (6 of 10)

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position.

- In unit-vector form, we write:

$$\vec{v} = \frac{d}{dt}\left(x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}\right) = \frac{dx}{dt}\hat{\mathrm{i}} + \frac{dy}{dt}\hat{\mathrm{j}} + \frac{dz}{dt}\hat{\mathrm{k}}.$$

- Which can also be written:

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}},$$ **Equation (4-11)**

4-2 Average Velocity and Instantaneous Velocity (7 of 10)

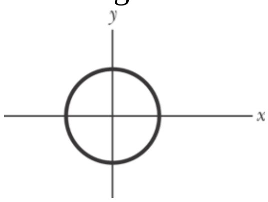
$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}.$$ **Equation (4-12)**

- Note: a velocity vector does not extend from one point to another, only shows direction and magnitude

4-2 Average Velocity and Instantaneous Velocity (8 of 10)

Checkpoint 1

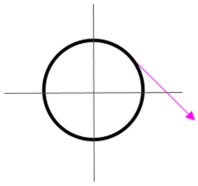
The figure shows a circular path taken by a particle. If the instantaneous velocity of the particle is $\vec{v} = (2\text{ m/s})\hat{i} - (2\text{ m/s})\hat{j}$, through which quadrant is the particle moving at that instant if it is traveling (a) clockwise and (b) counterclockwise around the circle? For both cases, draw $\vec{v}$ on the figure.



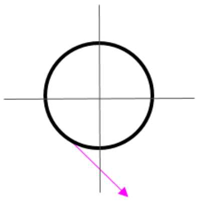
4-2 Average Velocity and Instantaneous Velocity (9 of 10)

Answer:

(a) Quadrant I



(b) Quadrant III



4-2 Avg. Velocity and Inst. Velocity (10 of 10)

Two-dimensional velocity, rabbit run

For the rabbit in the preceding Sample Problem, find the velocity $\vec{v}$ at time $t = 15$ s.

KEY IDEA

We can find $\vec{v}$ by taking derivatives of the components of the rabbit's position vector.

Calculations: Applying the v_x part of Eq. 4-12 to Eq. 4-5, we find the x component of $\vec{v}$ to be

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28)$$
$$= -0.62t + 7.2. \qquad (4\text{-}13)$$

At $t = 15$ s, this gives $v_x = -2.1$ m/s. Similarly, applying the v_y part of Eq. 4-12 to Eq. 4-6, we find

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30)$$
$$= 0.44t - 9.1. \qquad (4\text{-}14)$$

At $t = 15$ s, this gives $v_y = -2.5$ m/s. Equation 4-11 then yields

$$\vec{v} = (-2.1 \text{ m/s})\hat{i} + (-2.5 \text{ m/s})\hat{j}, \qquad \text{(Answer)}$$

which is shown in Fig. 4-5, tangent to the rabbit's path and in the direction the rabbit is running at $t = 15$ s.

To get the magnitude and angle of $\vec{v}$, either we use a vector-capable calculator or we follow Eq. 3-6 to write

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-2.1 \text{ m/s})^2 + (-2.5 \text{ m/s})^2}$$
$$= 3.3 \text{ m/s} \qquad \text{(Answer)}$$

and
$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-2.5 \text{ m/s}}{-2.1 \text{ m/s}}\right)$$
$$= \tan^{-1} 1.19 = -130°. \qquad \text{(Answer)}$$

Check: Is the angle $-130°$ or $-130° + 180° = 50°$?

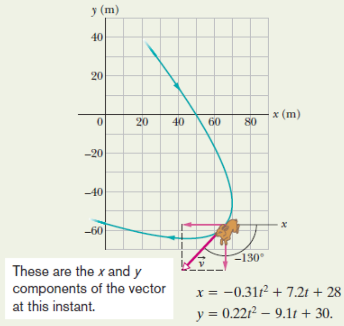


Fig. 4-5 The rabbit's velocity $\vec{v}$ at $t = 15$ s.

4-3 Average Acceleration and Instantaneous Acceleration (1 to 8)

Learning Objectives

4.08 Identify that acceleration is a vector quantity, and thus has both magnitude and direction.

4.09 Draw two-dimensional and three-dimensional acceleration vectors for a particle, indicating the components.

4.10 Given the initial and final velocity vectors of a particle and the time interval, determine the average acceleration vector.

4.11 Given a particle's velocity vector as a function of time, determine its (instantaneous) acceleration vector.

4.12 For each dimension of motion, apply the constant-acceleration equations (Chapter 2) to relate acceleration, velocity, position, and time.

4-3 Average Acceleration and Instantaneous Acceleration (2 to 8)

- **Average acceleration** is
  - A change in velocity divided by its time interval

$$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}.$$

Equation (4-15)

- **Instantaneous acceleration** is again the limit $t \rightarrow 0$:

$$\vec{a} = \frac{d\vec{v}}{dt}.$$

Equation (4-16)

4-3 Average Acceleration and Instantaneous Acceleration (3 to 8)

- We can write Eq. 4-16 in unit-vector form:

$$\begin{aligned} \vec{a} &= \frac{d}{dt}\left(v_x \hat{\mathrm{i}} + v_y \hat{\mathrm{j}} + v_z \mathrm{k}\right) \\ &= \frac{dv_x}{dt}\hat{\mathrm{i}} + \frac{dv_y}{dt}\hat{\mathrm{j}} + \frac{dv_z}{dt}\mathrm{k}. \end{aligned}$$

4-3 Average Acceleration and Instantaneous Acceleration (4 to 8)

- We can rewrite as:

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\mathrm{k},$$ **Equation (4-17)**

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad \text{and} \quad a_z = \frac{dv_z}{dt}.$$ **Equation (4-18)**

- To get the components of acceleration, we differentiate the components of velocity with respect to time

4-3 Average Acceleration and Instantaneous Acceleration (5 to 8)

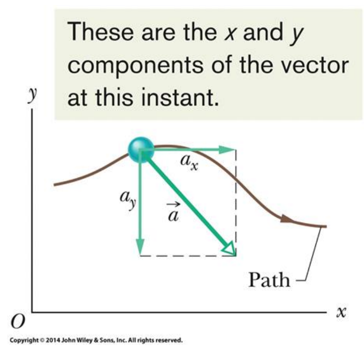


Figure 4-6

4-3 Average Acceleration and Instantaneous Acceleration (6 to 8)

- Note: as with velocity, an acceleration vector does not extend from one point to another, only shows direction and magnitude

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4-3 Average Acceleration and Instantaneous Acceleration (7 to 8)

Checkpoint 2

Here are four descriptions of the position (in meters) of a puck as it moves in an *xy* plane:

1. $x = -3t^2 + 4t - 2$ and $y = 6t^2 - 4t$
2. $x = -3t^3 - 4t$ and $y = -5t^2 + 6$
3. $\vec{r} = 2t^2\hat{\text{i}} - (4t+3)\hat{\text{j}}$
4. $\vec{r} = (4t^3 - 2t)\hat{\text{i}} + 3\hat{\text{j}}$

Answer:

1) x : yes, y : yes, a : yes
2) x : no, y : yes, a : no
3) x : yes, y : yes, a : yes
4) x : no, y : yes, a : no

Are the *x* and *y* acceleration components constant? Is acceleration $\vec{a}$ constant?

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4-3 Avg. Acceleration and Inst. Acceleration
(8 to 8)

Two-dimensional acceleration, rabbit run

For the rabbit in the preceding two Sample Problems, find the acceleration $\vec{a}$ at time $t = 15$ s.

KEY IDEA

We can find $\vec{a}$ by taking derivatives of the rabbit's velocity components.

Calculations: Applying the a_x part of Eq. 4-18 to Eq. 4-13, we find the x component of $\vec{a}$ to be

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62t + 7.2) = -0.62 \text{ m/s}^2.$$

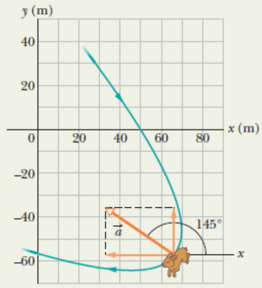
Similarly, applying the a_y part of Eq. 4-18 to Eq. 4-14 yields the y component as

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(0.44t - 9.1) = 0.44 \text{ m/s}^2.$$

We see that the acceleration does not vary with time (it is a constant) because the time variable t does not appear in the expression for either acceleration component. Equation 4-17 then yields

$$\vec{a} = (-0.62 \text{ m/s}^2)\hat{i} + (0.44 \text{ m/s}^2)\hat{j}, \quad \text{(Answer)}$$

which is superimposed on the rabbit's path in Fig. 4-7.



These are the x and y components of the vector at this instant.

Fig. 4-7 The acceleration $\vec{a}$ of the rabbit at $t = 15$ s. The rabbit happens to have this same acceleration at all points on its path.

4-4 Projectile Motion (1 of 15)

Learning Objectives

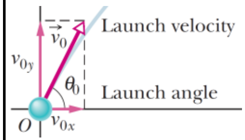
4.13 On a sketch of the path taken in projectile motion, explain the magnitudes and directions of the velocity and acceleration components during the flight.

4.14 Given the launch velocity in either magnitude-angle or unit-vector notation, calculate the particle's position, displacement, and velocity at a given instant during the flight.

4.15 Given data for an instant during the flight, calculate the launch velocity.

4-4 Projectile Motion (2 of 15)

- A **projectile** is
  - A particle moving in the vertical plane
  - With some initial velocity
  - Whose acceleration is always free-fall acceleration (g)
- The motion of a projectile is **projectile motion**
- Launched with an initial velocity v_0



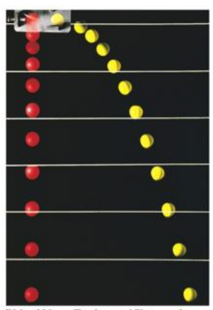
$$\vec{v}_0 = v_{0x}\hat{\mathrm{i}} + v_{0y}\hat{\mathrm{j}}. \qquad \textbf{Equation (4-19)}$$

$$v_{0x} = v_0\cos\theta_0 \quad \text{and} \quad v_{0y} = v_0\sin\theta_0. \qquad \textbf{Equation (4-20)}$$

In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

4-4 Projectile Motion (3 of 15)

- Therefore, we can decompose two-dimensional motion into 2 one-dimensional problems



Richard Megna/Fundamental Photographs

Figure 4-10

4-4 Projectile Motion (4 of 15)

Checkpoint 3

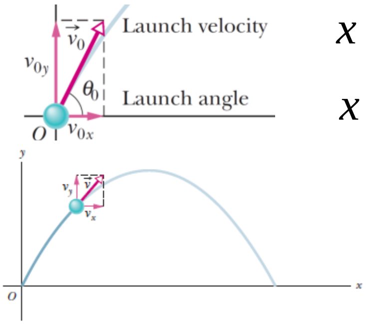
At a certain instant, a fly ball has velocity $\vec{v} = 25\hat{i} - 4.9\hat{j}$ (the *x* axis is horizontal, the *y* axis is upward, and $\vec{v}$ is in meters per second). Has the ball passed its highest point?

Answer:

Yes. The y-velocity is negative, so the ball is now falling.

4-4 Projectile Motion (5 of 15)

- Horizontal motion:
  - No acceleration, so velocity is constant (recall Eq. 2-15):

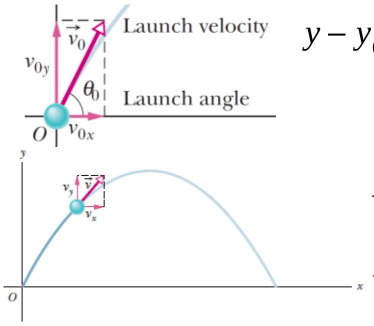


$$x - x_0 = v_{0x}t.$$

$$x - x_0 = (v_0 \cos\theta_0)t. \quad \textbf{Equation (4-21)}$$

4-4 Projectile Motion (6 of 15)

- Vertical motion:
  - Acceleration is always -g (recall Eqs. 2-15, 2-11, 2-16):



$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2$$

$$= (v_0 \sin\theta_0)t - \frac{1}{2}gt^2, \quad \textbf{Eq. (4-22)}$$

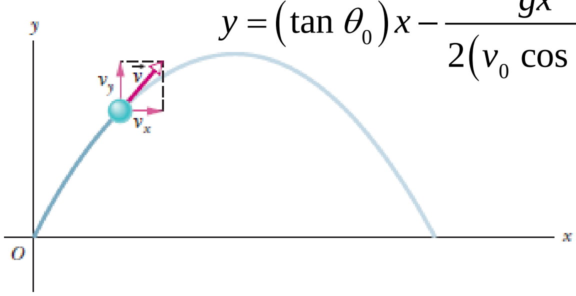
$$v_y = v_0 \sin\theta_0 - gt \quad \textbf{Eq. (4-23)}$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0). \quad \textbf{Eq. (4-24)}$$

4-4 Projectile Motion (7 of 15)

- The projectile's **trajectory** is
  - Its path through space (traces a parabola)
  - Found by eliminating time between Eqs. 4-21 and 4-22:

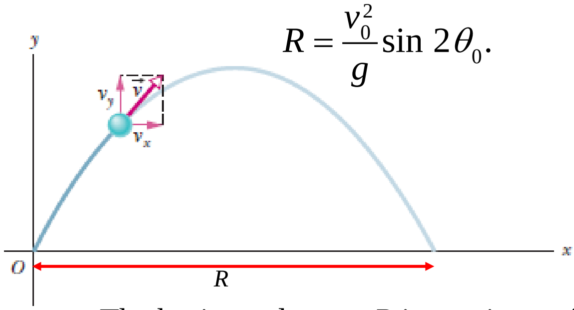
$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0 \cos\theta_0)^2} \quad \textbf{Equation (4-25)}$$



4-4 Projectile Motion (8 of 15)

- The **horizontal range** is:
  - The distance the projectile travels in *x* by the time it returns to its initial height

$$R = \frac{v_0^2}{g}\sin 2\theta_0. \qquad \textbf{Equation (4-26)}$$



The horizontal range *R* is maximum for a launch angle of 45°.

4-4 Projectile Motion (9 of 15)

Projectile dropped from airplane

In Fig. 4-14, a rescue plane flies at 198 km/h (= 55.0 m/s) and constant height h = 500 m toward a point directly over a victim, where a rescue capsule is to land.

(a) What should be the angle ϕ of the pilot's line of sight to the victim when the capsule release is made?

KEY IDEAS

Once released, the capsule is a projectile, so its horizontal and vertical motions can be considered separately (we need not consider the actual curved path of the capsule).

Calculations: In Fig. 4-14, we see that ϕ is given by

$$\phi = \tan^{-1}\frac{x}{h}, \qquad (4\text{-}27)$$

where x is the horizontal coordinate of the victim (and of the capsule when it hits the water) and h = 500 m. We should be able to find x with Eq. 4-21:

$$x - x_0 = (v_0 \cos\theta_0)t. \qquad (4\text{-}28)$$

Here we know that $x_0 = 0$ because the origin is placed at the point of release. Because the capsule is *released* and not shot from the plane, its initial velocity $\vec{v}_0$ is equal to the plane's velocity. Thus, we know also that the initial velocity has magnitude v_0 = 55.0 m/s and angle θ_0 = 0° (measured relative to the positive direction of the x axis). However, we do not know the time t the capsule takes to move from the plane to the victim.

To find t, we next consider the *vertical* motion and specifically Eq. 4-22:

$$y - y_0 = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2. \qquad (4\text{-}29)$$

Here the vertical displacement $y - y_0$ of the capsule is −500 m (the negative value indicates that the capsule moves *downward*). So,

$$-500 \text{ m} = (55.0 \text{ m/s})(\sin 0°)t - \tfrac{1}{2}(9.8 \text{ m/s}^2)t^2. \qquad (4\text{-}30)$$

Solving for t, we find t = 10.1 s. Using that value in Eq. 4-28 yields

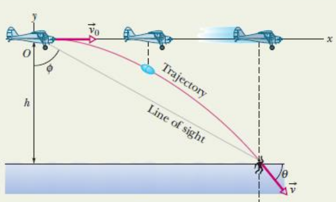
$$x - 0 = (55.0 \text{ m/s})(\cos 0°)(10.1 \text{ s}), \qquad (4\text{-}31)$$

or

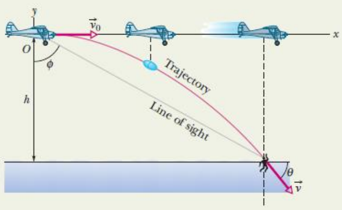
$$x = 555.5 \text{ m}.$$

Then Eq. 4-27 gives us

$$\phi = \tan^{-1}\frac{555.5 \text{ m}}{500 \text{ m}} = 48.0°. \qquad \text{(Answer)}$$



4-4 Projectile Motion (10 of 15)



(b) As the capsule reaches the water, what is its velocity $\vec{v}$ in unit-vector notation and in magnitude-angle notation?

KEY IDEAS

(1) The horizontal and vertical components of the capsule's velocity are independent. (2) Component v_x does not change from its initial value $v_{0x} = v_0 \cos \theta_0$ because there is no horizontal acceleration. (3) Component v_y changes from its initial value $v_{0y} = v_0 \sin \theta_0$ because there is a vertical acceleration.

Calculations: When the capsule reaches the water,

$$v_x = v_0 \cos \theta_0 = (55.0 \text{ m/s})(\cos 0°) = 55.0 \text{ m/s}.$$

Using Eq. 4-23 and the capsule's time of fall $t = 10.1$ s, we also find that when the capsule reaches the water,

$$\begin{aligned} v_y &= v_0 \sin \theta_0 - gt \qquad (4\text{-}32) \\ &= (55.0 \text{ m/s})(\sin 0°) - (9.8 \text{ m/s}^2)(10.1 \text{ s}) \\ &= -99.0 \text{ m/s}. \end{aligned}$$

Thus, at the water

$$\vec{v} = (55.0 \text{ m/s})\hat{i} - (99.0 \text{ m/s})\hat{j}. \qquad \text{(Answer)}$$

Using Eq. 3-6 as a guide, we find that the magnitude and the angle of $\vec{v}$ are

$$v = 113 \text{ m/s} \quad \text{and} \quad \theta = -60.9°. \qquad \text{(Answer)}$$

4-4 Projectile Motion (11 of 15)

Checkpoint 4

A fly ball is hit to the outfield. During its flight (ignore the effects of the air), what happens to its (a) horizontal and (b) vertical components of velocity? What are the (c) horizontal and (d) vertical components of its acceleration during ascent, during descent, and at the topmost point of its flight?

Answer:

(a) is unchanged

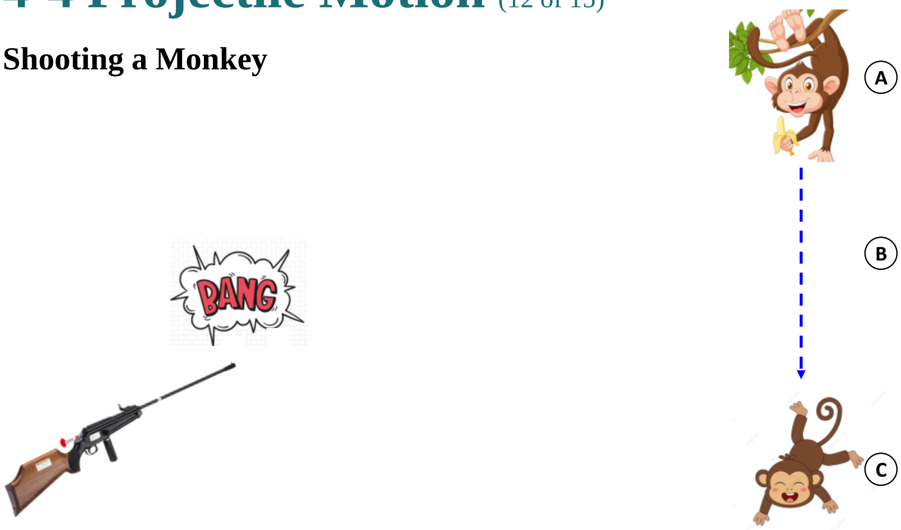
(b) decreases (becomes negative)

(c) 0 at all times

(d) $-g\left(-9.8 \text{ m/s}^2\right)$ at all times

4-4 Projectile Motion (12 of 15)

Shooting a Monkey

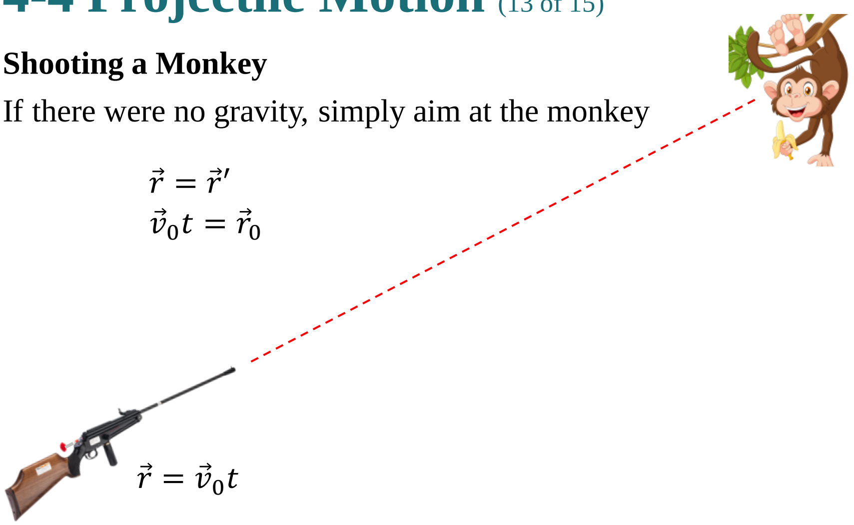


4-4 Projectile Motion (13 of 15)

$\vec{r}' = \vec{r}_0$

Shooting a Monkey

If there were no gravity, simply aim at the monkey



$$\vec{r} = \vec{r}'$$
$$\vec{v}_0 t = \vec{r}_0$$

$\vec{r} = \vec{v}_0 t$

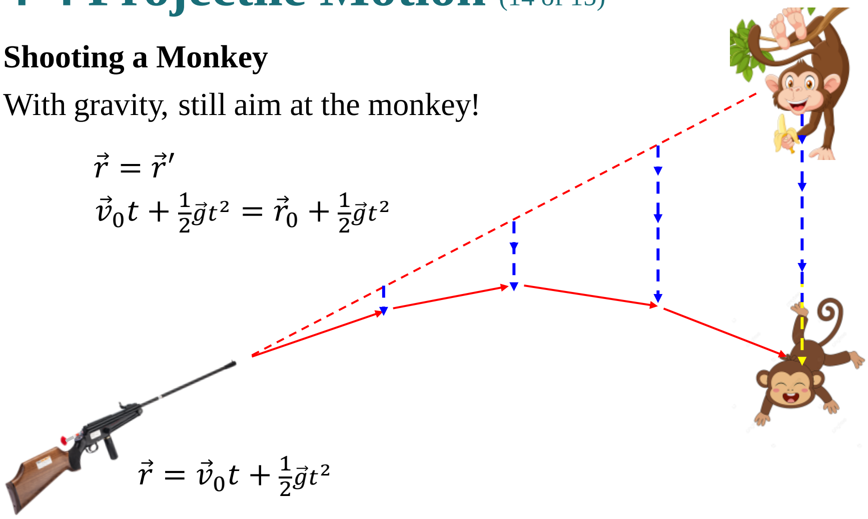
4-4 Projectile Motion (14 of 15)

$\vec{r}\,' = \vec{r}_0 + \frac{1}{2}\vec{g}t^2$

Shooting a Monkey

With gravity, still aim at the monkey!

$$\vec{r} = \vec{r}\,'$$
$$\vec{v}_0 t + \frac{1}{2}\vec{g}t^2 = \vec{r}_0 + \frac{1}{2}\vec{g}t^2$$



$\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{g}t^2$

4-4 Projectile Motion (15 of 15)

- In these calculations we assume air resistance is negligible
- In many situations this is a poor assumption:

Table 4-1 Two Fly Balls[a]

	Path I (Air)	Path II (Vacuum)
Range	98.5 m	177 m
Maximum height	53.0 m	76.8 m
Time of flight	6.6 s	7.9 s

[a]See Fig. 4-13. The launch angle is 60° and the launch speed is 44.7 m/s.

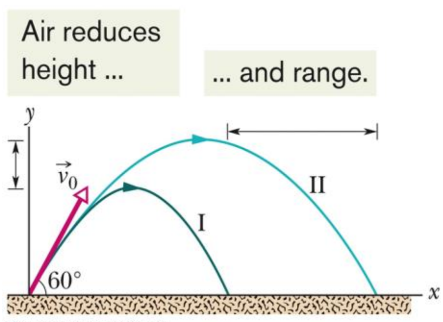


Figure 4-13

4 Summary (1 of 6)

Position Vector

- Locates a particle in 3-space

$$\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\mathrm{k}, \qquad \textbf{Equation (4-1)}$$

Displacement

- Change in position vector

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1. \qquad \textbf{Equation (4-2)}$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{\mathrm{i}} + (y_2 - y_1)\hat{\mathrm{j}} + (z_2 - z_1)\mathrm{k}, \qquad \textbf{Equation (4-3)}$$

$$\Delta\vec{r} = \Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\mathrm{k}. \qquad \textbf{Equation (4-4)}$$

4 Summary (2 of 6)

Average and Instantaneous Velocity

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t}. \qquad \textbf{Equation (4-8)}$$

$$\vec{v} = \frac{d\vec{r}}{dt}. \qquad \textbf{Equation (4-10)}$$

Average and Instantaneous Accel.

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}. \qquad \textbf{Equation (4-15)}$$

$$\vec{a} = \frac{d\vec{v}}{dt}. \qquad \textbf{Equation (4-16)}$$

4 Summary (3 of 6)

Projectile Motion

- Flight of particle subject only to free-fall acceleration (g)

$$y - y_0 = v_{0y} t - \frac{1}{2} g t^2 = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2,$$ **Equation (4-22)**

$$v_y = v_0 \sin \theta_0 - gt$$ **Equation (4-23)**

- Trajectory is parabolic path

$$y = (\tan \theta_0) x - \frac{g x^2}{2 (v_0 \cos \theta_0)^2}$$ **Equation (4-25)**

4 Summary (4 of 6)

- Horizontal range:

$$R = \frac{v_0^2}{g} \sin 2\theta_0.$$ **Equation (4-26)**

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