Fundamentals Physics

Eleventh Edition

Halliday

Chapter 4
Motion in Two and Three Dimensions
part 2

2

4-5 Uniform Circular Motion (1 of 7)

Learning Objectives

4.16 Sketch the path taken in uniform circular motion and explain the velocity and acceleration vectors (magnitude and direction) during the motion.

4.17 Apply the relationships between the radius of the circular path, the period, the particle's speed, and the particle's acceleration magnitude.

 3

3

4-5 Uniform Circular Motion (2 of 7)

- A particle is in **uniform circular motion** if
  - It travels around a circle or circular arc
  - At a constant speed
- Since the velocity changes, the particle is accelerating!
- Velocity and acceleration have:
  - Constant magnitude
  - Changing direction

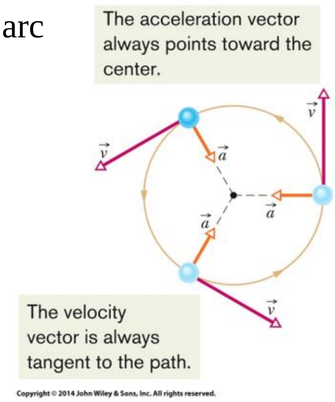
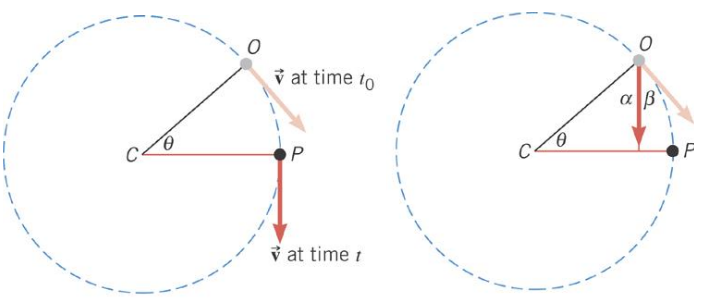


Figure 4-16

4

4-5 Uniform Circular Motion (3 of 7)

- Since the velocity changes, the particle is accelerating!
  - Constant radius R
  - Constant speed $v = |\vec{v}|$



$$\alpha + \beta = 90^\circ$$
$$\alpha + \theta = 90^\circ$$
$$\Downarrow$$
$$\beta = \theta$$

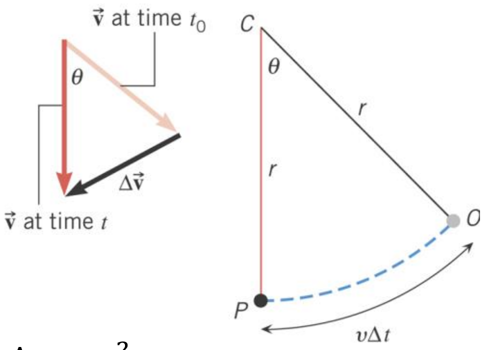
5

4-5 Uniform Circular Motion (4 of 7)

Similar triangles: $\frac{\Delta v}{v} = \frac{\Delta R}{R}$

But $\Delta R = v\Delta t$ for small Δt

So: $\frac{\Delta v}{v} = \frac{v\Delta t}{R}$ ➡ $\frac{\Delta v}{\Delta t} = \frac{v^2}{R}$



Hence the "centripetal" acceleration is,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{R}$$

Which points towards the center of the circle of radius R

4-5 Uniform Circular Motion (5 of 7)

- Acceleration is called **centripetal acceleration**
  - Means "center seeking"
  - Directed radially inward

$$a = \frac{v^2}{r}$$ **Equation (4-34)**

- The **period of revolution** is:
  - The time it takes for the particle go around the closed path exactly once

$$T = \frac{2\pi r}{v}$$ **Equation (4-35)**

4-5 Uniform Circular Motion (6 of 7)

Checkpoint 5

An object moves at constant speed along a circular path in a horizontal *xy* plane, with the center at the origin. When the object is at $x = -2$ m, its velocity is $-(4 \text{ m/s})\hat{j}$.

Give the object's (a) velocity and (b) acceleration at $y = 2$ m.

Answer:

(a) $-(4 \text{ m/s})i$

(b) $-(8 \text{ m/s}^2)\,j$

4-5 Uniform Circular Motion (7 of 7)

Top gun pilots in turns

"Top gun" pilots have long worried about taking a turn too tightly. As a pilot's body undergoes centripetal acceleration, with the head toward the center of curvature, the blood pressure in the brain decreases, leading to loss of brain function.

There are several warning signs. When the centripetal acceleration is $2g$ or $3g$, the pilot feels heavy. At about $4g$, the pilot's vision switches to black and white and narrows to "tunnel vision." If that acceleration is sustained or increased, vision ceases and, soon after, the pilot is unconscious—a condition known as *g*-LOC for "*g*-induced loss of consciousness."

What is the magnitude of the acceleration, in *g* units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\vec{v}_i = (400\hat{i} + 500\hat{j})$ m/s and 24.0 s later leaves the turn with a velocity of $\vec{v}_f = (-400\hat{i} - 500\hat{j})$ m/s?

KEY IDEAS

We assume the turn is made with uniform circular motion. Then the pilot's acceleration is centripetal and has magnitude a given by Eq. 4-34 ($a = v^2/R$), where R is the circle's radius. Also, the time required to complete a full circle is the period given by Eq. 4-35 ($T = 2\pi R/v$).

Calculations: Because we do not know radius R, let's solve Eq. 4-35 for R and substitute into Eq. 4-34. We find

$$a = \frac{2\pi v}{T}.$$

Speed v here is the (constant) magnitude of the velocity during the turning. Let's substitute the components of the initial velocity into Eq. 3-6:

$$v = \sqrt{(400 \text{ m/s})^2 + (500 \text{ m/s})^2} = 640.31 \text{ m/s}.$$

To find the period T of the motion, first note that the final velocity is the reverse of the initial velocity. This means the aircraft leaves on the opposite side of the circle from the initial point and must have completed half a circle in the given 24.0 s. Thus a full circle would have taken $T = 48.0$ s. Substituting these values into our equation for a, we find

$$a = \frac{2\pi(640.31 \text{ m/s})}{48.0 \text{ s}} = 83.81 \text{ m/s}^2 \approx 8.6g. \quad \text{(Answer)}$$

4-6 Relative Motion in One Dimension (1 of 6)

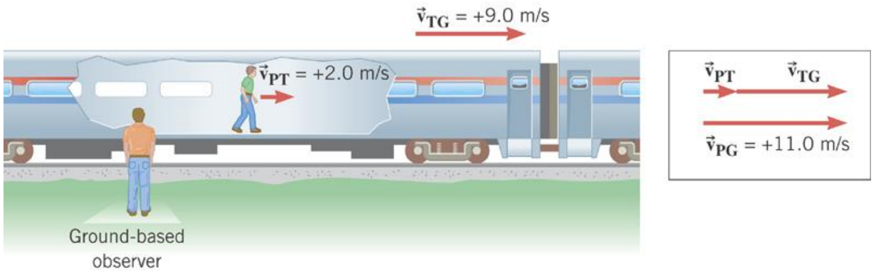
Learning Objectives

4.18 Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference frames that move relative to each other at a constant velocity and along a single axis.

4-6 Relative Motion in One Dimension (2 of 6)

Relative Velocity

Observed velocity depends on velocity of observer!



4-6 Relative Motion in One Dimension (3 of 6)

- Measures of position and velocity depend on the **reference frame** of the measurer
  - How is the observer moving?
  - Our usual reference frame is that of the ground
- Read subscripts
  - "*PA*" as "*P* as measured by *A*"
  - "*PB*" as "*P* as measured by *B*",
  - "*BA*" as "*B* as measured by *A*"
- Frames *A* and *B* are each watching the movement of object *P*

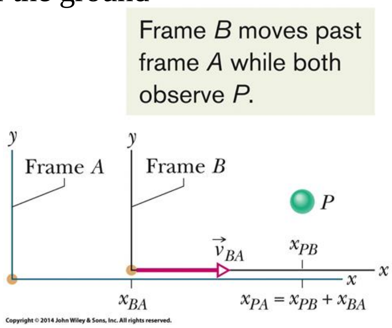


Figure 4-18

4-6 Relative Motion in One Dimension (4 of 6)

- Positions in different frames are related by:

$$x_{PA} = x_{PB} + x_{BA}.$$ **Equation (4-40)**

- Taking the derivative, we see velocities are related by:

$$\frac{d}{dt}(x_{PA}) = \frac{d}{dt}(x_{PB}) + \frac{d}{dt}(x_{BA}).$$

$$v_{PA} = v_{PB} + v_{BA}.$$ **Equation (4-41)**

4-6 Relative Motion in One Dimension (5 of 6)

- But accelerations (for non-accelerating reference frames, $a_{BA} = 0$) are related by

$$\frac{d}{dt}(v_{PA}) = \frac{d}{dt}(v_{PB}) + \frac{d}{dt}(v_{BA}).$$

$$a_{PA} = a_{PB}. \qquad \textbf{Equation (4-42)}$$

4-6 Relative Motion in One Dimension (6 of 6)

Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration for a moving particle.

Example

Frame A: $x = 2$ m, $v = 4$ m/s
Frame B: $x = 3$ m, $v = -2$ m/s
P as measured by A: $x_{PA} = 5$ m, $v_{PA} = 2$ m/s, $a = 1$ m/s^2

So P as measured by B:

- $x_{PB} = x_{PA} + x_{AB} = 5$ m + (2m – 3m) = 4 m
- $v_{PB} = v_{PA} + v_{AB} = 2$ m/s + (4 m/s – −2m/s) = 8 m/s
- $a = 1\ m/s^2$

4-7 Relative Motion in Two Dimensions (1 of 10)

Learning Objectives

4.19 Apply the relationship between a particle's position, velocity, and acceleration as measured from two reference frames that move relative to each other at a constant velocity and in two dimensions.

4-7 Relative Motion in Two Dimensions (2 of 10)

- Frames *A* and *B* are both observing the motion of *P*

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

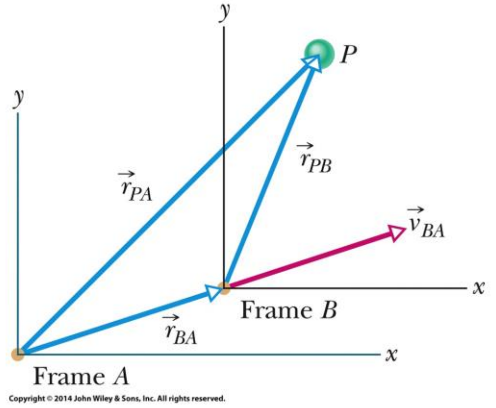


Figure 4-19

4-7 Relative Motion in Two Dimensions (3 of 10)

- The same as in one dimension, but now with vectors:
- Positions in different frames are related by:

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}.$$ **Equation (4-43)**

- Velocities:

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}.$$ **Equation (4-44)**

- Accelerations (for non-accelerating reference frames):

$$\vec{a}_{PA} = \vec{a}_{PB}.$$ **Equation (4-45)**

- Again, observers in different frames will see the same acceleration

 18

18

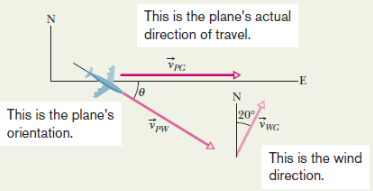
4-7 Relative Motion in Two Dimensions (4 of 10)

Relative motion, two dimensional, airplanes

In Fig. 4-20*a*, a plane moves due east while the pilot points the plane somewhat south of east, toward a steady wind that blows to the northeast. The plane has velocity $\vec{v}_{PW}$ relative to the wind, with an airspeed (speed relative to the wind) of 215 km/h, directed at angle θ south of east. The wind has velocity $\vec{v}_{WG}$ relative to the ground with speed 65.0 km/h, directed 20.0° east of north. What is the magnitude of the velocity $\vec{v}_{PG}$ of the plane relative to the ground, and what is θ?

KEY IDEAS

The situation is like the one in Fig. 4-19. Here the moving particle *P* is the plane, frame *A* is attached to the ground (call it *G*), and frame *B* is "attached" to the wind (call it *W*). We need a vector diagram like Fig. 4-19 but with three velocity vectors.



Calculations: First we construct a sentence that relates the three vectors shown in Fig. 4-20*b*:

velocity of plane relative to ground (*PG*) = velocity of plane relative to wind (*PW*) + velocity of wind relative to ground. (*WG*)

This relation is written in vector notation as

$$\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}. \tag{4-46}$$

We need to resolve the vectors into components on the coordinate system of Fig. 4-20*b* and then solve Eq. 4-46 axis by axis. For the *y* components, we find

$$v_{PG,y} = v_{PW,y} + v_{WG,y}$$

or $\quad 0 = -(215 \text{ km/h}) \sin\theta + (65.0 \text{ km/h})(\cos 20.0°).$

Solving for θ gives us

$$\theta = \sin^{-1}\frac{(65.0 \text{ km/h})(\cos 20.0°)}{215 \text{ km/h}} = 16.5°. \qquad \text{(Answer)}$$

19

4-7 Relative Motion in Two Dimensions (5 of 10)

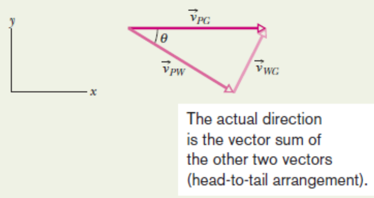
Relative motion, two dimensional, airplanes

Similarly, for the *x* components we find

$$v_{PG,x} = v_{PW,x} + v_{WG,x}.$$

Here, because $\vec{v}_{PG}$ is parallel to the *x* axis, the component $v_{PG,x}$ is equal to the magnitude v_{PG}. Substituting this notation and the value $\theta = 16.5°$, we find

$$v_{PG} = (215 \text{ km/h})(\cos 16.5°) + (65.0 \text{ km/h})(\sin 20.0°)$$
$$= 228 \text{ km/h}. \quad \text{(Answer)}$$

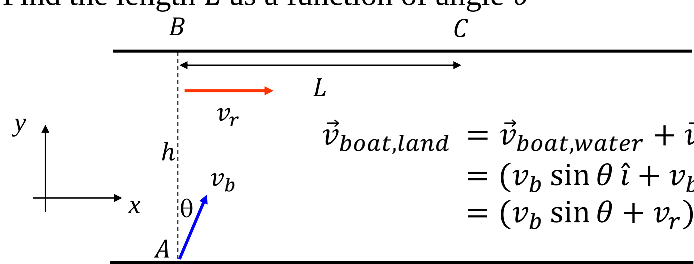


 20

4-7 Relative Motion in Two Dimensions (6 of 10)

Crossing the River

Find the length L as a function of angle θ



$$\vec{v}_{boat,land} = \vec{v}_{boat,water} + \vec{v}_{water,land}$$
$$= (v_b \sin\theta\,\hat{\imath} + v_b \cos\theta\,\hat{\jmath}) + v_r\hat{\imath}$$
$$= (v_b \sin\theta + v_r)\hat{\imath} + v_b\cos\theta\,\hat{\jmath}$$

The time needed for the crossing is $h = v_b \cos\theta\, t \rightarrow t = \dfrac{d}{v_b \cos\theta}$

Hence the distance from B of landing side is

$$L(\theta) = (v_b \sin\theta + v_r)t = \frac{h(v_b \sin\theta + v_r)}{v_b \cos\theta}$$

 21

4-7 Relative Motion in Two Dimensions (7 of 10)

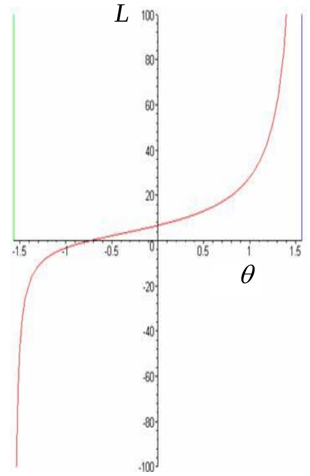
Crossing the River

With $v_b = 3\ m/s$, $v_r = 2\ m/s$ $(v_b > v_r)$ and $h = 10\ m$, we obtain the graphs of length L with respect to θ as shown on the right.

We see that $L = 0$ can be obtained, and the boat should be directed to a certain angle that is calculated as follows

$$L(\theta) = \frac{h(v_b \sin\theta + v_r)}{v_b \cos\theta} = 0$$

$$v_b \sin\theta + v_r = 0 \quad \rightarrow \quad \sin\theta = -\frac{v_r}{v_b}$$



 22

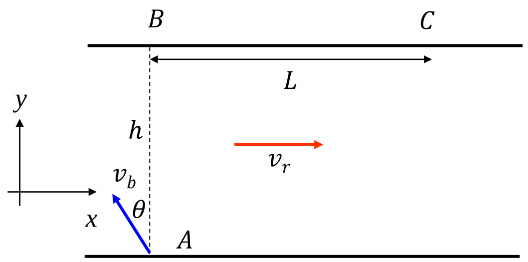
22

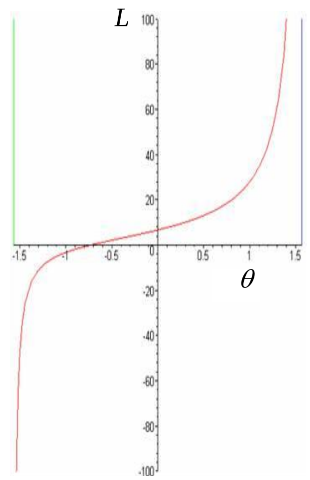
4-7 Relative Motion in Two Dimensions (8 of 10)

Crossing the River

$$v_b \sin\theta + v_r = 0 \quad \rightarrow \quad \sin\theta = -\frac{v_r}{v_b}$$

Yielding the value of $\theta = -41.8^{\circ}$, which means that the boat is directed with an angle to the left of the AB line.





 23

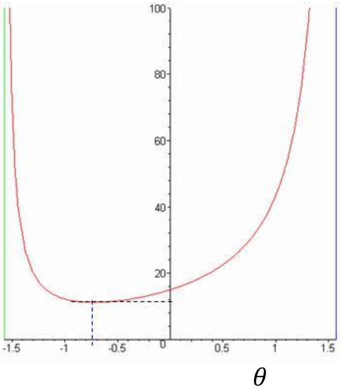
23

4-7 Relative Motion in Two Dimensions (9 of 10)

Crossing the River

If $v_b < v_r$ for example $v_r = 3\ m/s$, $v_b = 2\ m/s$ ($v_b < v_r$) and $h = 10\ m$, we obtain the graphs of length L with respect to θ as shown on the right.

Note that L will never be zero. BUT there is a minimum value of L corresponding to a specific angle. This angle can be found by finding the minimum of L



$$\frac{dL}{d\theta} = \frac{h\left(v_b^2 \cos^2\theta + v_b^2 \sin^2\theta + v_b v_r \sin\theta\right)}{v_b^2 \cos^2\theta} = \frac{h\left(v_b^2 + v_b v_r \sin\theta\right)}{v_b^2 \cos^2\theta}$$

 24

24

4-7 Relative Motion in Two Dimensions(10 of 10)

Crossing the River

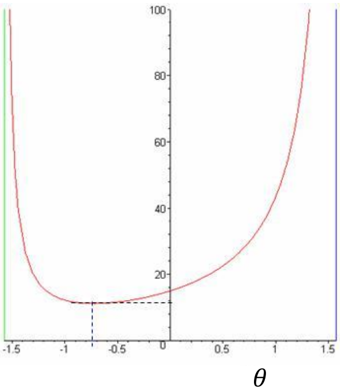
$$\frac{dL}{d\theta} = \frac{h\left(v_b^2 + v_b v_r \sin\theta\right)}{v_b^2 \cos^2\theta} \rightarrow \frac{dL}{d\theta} = 0$$

yields

$$v_b^2 + v_b v_r \sin\theta = 0 \rightarrow \sin\theta = -\frac{v_b}{v_r}$$

For the above data, we found that $\theta = -41.8^{\circ}$, which means that the boat should be directed to the left of the AB line, and the minimum distance is

$$L = 11.18\ \text{m}$$



 25

25

4 Summary (5 of 6)

Uniform Circular Motion

- Magnitude of acceleration:

$$a = \frac{v^2}{r}$$ **Equation (4-34)**

- Time to complete a circle:

$$T = \frac{2\pi r}{v}$$ **Equation (4-35)**

4 Summary (6 of 6)

Relative Motion

- For non-accelerating reference frames

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}.$$ **Equation (4-44)**

$$\vec{a}_{PA} = \vec{a}_{PB}.$$ **Equation (4-45)**

Copyright