Fundamentals Physics

Eleventh Edition

Halliday

Chapter 10

Rotation

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10-1 Rotational Variables (1 of 13)

Learning Objectives

10.01 Identify that if all parts of a body rotate around a fixed axis locked together, the body is a rigid body.

10.02 Identify that the angular position of a rotating rigid body is the angle that an internal reference line makes with a fixed, external reference line.

10.03 Apply the relationship between angular displacement and the initial and final angular positions.

10.04 Apply the relationship between average angular velocity, angular displacement, and the time interval for that displacement.

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10-1 Rotational Variables (2 of 13)

10.05 Apply the relationship between average angular acceleration, change in angular velocity, and the time interval for that change.

10.06 Identify that counterclockwise motion is in the positive direction and clockwise motion is in the negative direction.

10.07 Given angular position as a function of time, calculate the instantaneous angular velocity at any particular time and the average angular velocity between any two particular times.

10.08 Given a graph of angular position versus time, determine the instantaneous angular velocity at a particular time and the average angular velocity between any two particular times.

10-1 Rotational Variables (3 of 13)

10.09 Identify instantaneous angular speed as the magnitude of the instantaneous angular velocity.

10.10 Given angular velocity as a function of time, calculate the instantaneous angular acceleration at any particular time and the average angular acceleration between any two particular times.

10.11 Given a graph of angular velocity versus time, determine the instantaneous angular acceleration at any particular time and the average angular acceleration between any two particular times.

10-1 Rotational Variables (4 of 13)

10.12 Calculate a body's change in angular velocity by integrating its angular acceleration function with respect to time.

10.13 Calculate a body's change in angular position by integrating its angular velocity function with respect to time.

10-1 Rotational Variables (5 of 13)

- We now look at motion of **rotation**
- We will find the same laws apply
- But we will need new quantities to express them
  - Torque
  - Rotational inertia
- A **rigid body** rotates as a unit, locked together
- We look at rotation about a **fixed axis**
- These requirements exclude from consideration:
  - The Sun, where layers of gas rotate separately
  - A rolling bowling ball, where rotation and translation occur

10-1 Rotational Variables (6 of 13)

- The fixed axis is called the **axis of rotation**
- Figs 10-2, 10-3 show a reference line
- The **angular position** of this line (and of the object) is taken relative to a fixed direction, the **zero angular position**

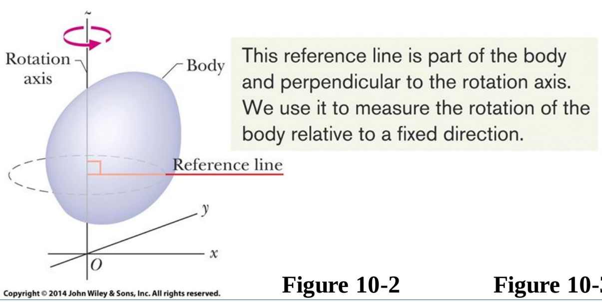


Figure 10-2

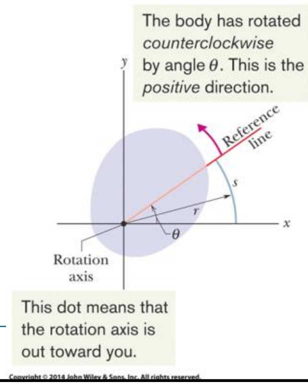


Figure 10-3

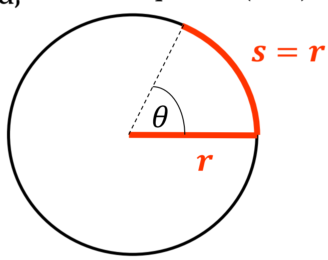
10-1 Rotational Variables (7 of 13)

- Measure using **radians** (rad): dimensionless

$$\theta = \frac{s}{r} \quad \text{(radian measure).}$$ **Equation (10-1)**

$$1 \text{ rev} = 360° = \frac{2\pi r}{r} = 2\pi \text{ rad},$$ **Equation (10-2)**

- Take a line of length r and stretch it along the edge of the circle, the angle swept out is 1 radian.



$$\pi \text{ radians} = 180°$$

- Do not reset θ to zero after a full rotation.

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10-1 Rotational Variables (8 of 13)

- We know all there is to know about the kinematics of rotation if we have $\theta(t)$ for an object.
- Define angular displacement as:

$$\Delta\theta = \theta_2 - \theta_1. \qquad \textbf{Equation (10-4)}$$

- "Clocks are negative":

An angular displacement in the counterclockwise direction is positive, and one in the clockwise direction is negative.

10-1 Rotational Variables (9 of 13)

Checkpoint 1

A disk can rotate about its central axis like a merry-go-round. Which of the following pairs of values for its initial and final angular positions, respectively, give a negative angular displacement: (a) −3 rad, +5 rad, (b) −3 rad, −7 rad, (c) 7 rad, −3 rad?

Answer:

Choices (b) and (c)

10-1 Rotational Variables (10 of 13)

- **Average angular velocity**: angular displacement during a time interval

$$\omega_{\text{avg}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}, \qquad \textbf{Equation (10-5)}$$

- **Instantaneous angular velocity**: limit as $\Delta t \to 0$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}. \qquad \textbf{Equation (10-6)}$$

- If the body is rigid, these equations hold for all points on the body
- Magnitude of angular velocity = **angular speed**

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10-1 Rotational Variables (11 of 13)

- Figure 10-4 shows the values for a calculation of average angular velocity

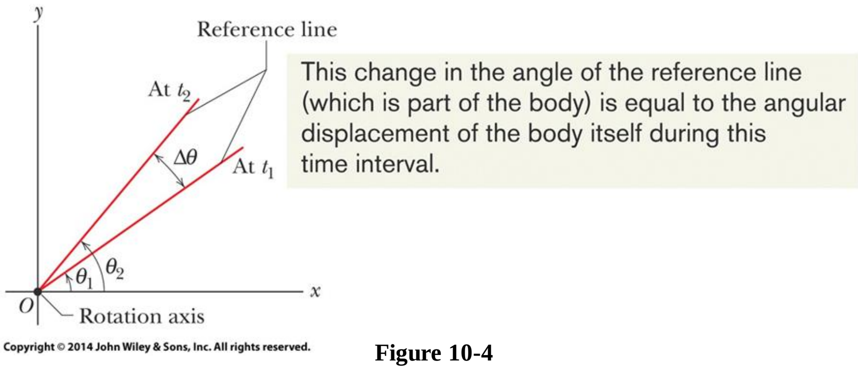

Figure 10-4

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10-1 Rotational Variables (12 of 13)

- **Average angular acceleration:** angular velocity change during a time interval

$$\alpha_{\text{avg}} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t},$$ **Equation (10-7)**

- **Instantaneous angular velocity:** limit as $\Delta t \to 0$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}.$$ **Equation (10-8)**

10-1 Rotational Variables (13 of 13)

- If the body is rigid, these equations hold for all points on the body
- With right-hand rule to determine direction, angular velocity & acceleration can be written as vectors
- If the body rotates around the vector, then the vector points along the axis of rotation
- Angular displacements are not vectors, because the order of rotation matters for rotations around different axes

10-2 Rotation with Constant Angular Acceleration (1 of 9)

Learning Objectives

10.14 For constant angular acceleration, apply the relationships between angular position, angular displacement, angular velocity, angular acceleration, and elapsed time (Table 10-1).

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10-2 Rotation with Constant Angular Acceleration (2 of 9)

- The same equations hold as for constant linear acceleration, see Table 10-1
- We simply change linear quantities to angular ones
- Equations. 10-12 and 10-13 are the basic equations: all others can be derived from them

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10-2 Rotation with Constant Angular Acceleration (3 of 9)

Table 10-1 Equations of Motion for Constant Linear Acceleration and for Constant Angular Acceleration

Equation Number	Linear Equation	Missing Variable	Missing Variable	Angular Equation	Equation Number
(2-11)	$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$	(10-12)
(2-15)	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$	(10-13)
(2-16)	$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	(10-14)
(2-17)	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$	(10-15)
(2-18)	$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$	(10-16)

10-2 Rotation with Constant Angular Acceleration (4 of 9)

Checkpoint 2

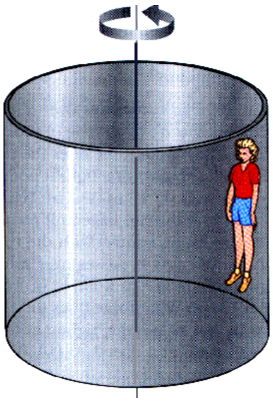
In four situations, a rotating body has angular position $\theta(t)$ given by (a) $\theta = 3t - 4$, (b) $\theta = -5t^3 + 4t^2 + 6$, (c) $\theta = \frac{2}{t^2} - \frac{4}{t}$, and (d) $\theta = 5t^2 - 3$. To which situations do the angular equations of Table 10-1 apply?

Answer:

Situations (a) and (d); the others do not have constant angular acceleration

10-2 Rotation with Constant Angular Acceleration (5 of 9)

Constant angular acceleration, riding a Rotor



10-2 Rotation with Constant Angular Acceleration (6 of 9)

Constant angular acceleration, riding a Rotor

While you are operating a Rotor (a large, vertical, rotating cylinder found in amusement parks), you spot a passenger in acute distress and decrease the angular velocity of the cylinder from 3.40 rad/s to 2.00 rad/s in 20.0 rev, at constant angular acceleration. (The passenger is obviously more of a "translation person" than a "rotation person.")

(a) What is the constant angular acceleration during this decrease in angular speed?

Calculations: Let's first do a quick check to see if we can solve the basic equations. The initial angular velocity is $\omega_0 = 3.40$ rad/s,

10-2 Rotation with Constant Angular Acceleration (7 of 9)

Constant angular acceleration, riding a Rotor

the angular displacement is $\theta - \theta_0 = 20.0$ rev, and the angular velocity at the end of that displacement is $\omega = 2.00$ rad/s. In addition to the angular acceleration α that we want, both basic equations also contain time t, which we do not necessarily want.

To eliminate the unknown t, we use Eq. 10-12 to write

$$t = \frac{\omega - \omega_0}{\alpha},$$

which we then substitute into Eq. 10-13 to write

$$\theta - \theta_0 = \omega_0\left(\frac{\omega - \omega_0}{\alpha}\right) + \tfrac{1}{2}\alpha\left(\frac{\omega - \omega_0}{\alpha}\right)^2.$$

10-2 Rotation with Constant Angular Acceleration (8 of 9)

Constant angular acceleration, riding a Rotor

Solving for α, substituting known data, and converting 20 rev to 125.7 rad, we find

$$\alpha = \frac{\omega^2 - \omega_0^2}{2(\theta - \theta_0)} = \frac{(2.00\text{ rad/s})^2 - (3.40\text{ rad/s})^2}{2(125.7\text{ rad})}$$

$$= -0.0301\text{ rad/s}^2. \qquad \text{(Answer)}$$

10-2 Rotation with Constant Angular Acceleration (9 of 9)

Constant angular acceleration, riding a Rotor

(b) How much time did the speed decrease take?

Calculation: Now that we know α, we can use Eq. 10-12 to solve for t:

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{2.00 \text{ rad/s} - 3.40 \text{ rad/s}}{-0.0301 \text{ rad/s}^2}$$

$$= 46.5 \text{ s.} \qquad \text{(Answer)}$$

10-3 Relating the Linear and Angular Variables (1 of 13)

Learning Objectives

10.15 For a rigid body rotating about a fixed axis, relate the angular variables of the body (angular position, angular velocity, and angular acceleration) and the linear variables of a particle on the body (position, velocity, and acceleration) at any given radius.

10.16 Distinguish between tangential acceleration and radial acceleration and draw a vector for each in a sketch of a particle on a body rotating about an axis, for both an increase in angular speed and a decrease.

10-3 Relating the Linear and Angular Variables (2 of 13)

- Linear and angular variables are related by r, perpendicular distance from the rotational axis
- Position (note θ must be in radians):

$$s = \theta r \qquad \textbf{Equation (10-17)}$$

- Speed (note ω must be in radian measure):

$$v = \omega r \qquad \textbf{Equation (10-18)}$$

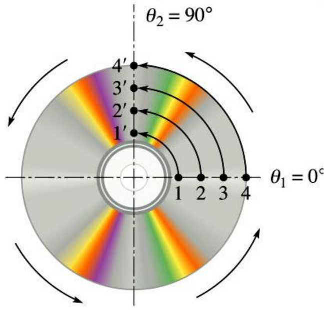
- We can express the period in radian measure:

$$T = \frac{2\pi}{\omega} \qquad \textbf{Equation (10-20)}$$

10-3 Relating the Linear and Angular Variables (3 of 13)

Example: A CD spins with angular frequency 20 radians/second. What is the linear speed at position 6 cm and 2 cm from the center of the CD?

$$v_6 = \omega r_6 = (20)(0.06) = 1.2 m/s$$
$$v_2 = \omega r_2 = (20)(0.02) = 0.4 m/s$$



(a)

10-3 Relating the Linear and Angular Variables (4 of 13)

- Tangential acceleration (radians):

$$a_t = \alpha r$$ **Equation (10-22)**

- We can write the radial acceleration in terms of angular velocity (radians):

$$a_r = \frac{v^2}{r} = \omega^2 r$$ **Equation (10-23)**

10-3 Relating the Linear and Angular Variables (5 of 13)

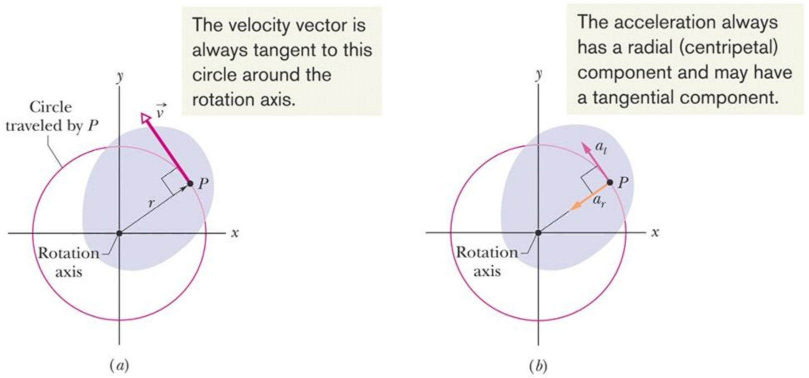


Figure 10-9

10-3 Relating the Linear and Angular Variables (6 of 13)

Checkpoint 3

A cockroach rides the rim of a rotating merry-go-round.
If the angular speed of this system (merry-go-round + cockroach) is constant, does the cockroach have (a) radial acceleration and (b) tangential acceleration?

If ω is decreasing, does the cockroach have (c) radial acceleration and (d) tangential acceleration?

Answer:

(a) yes

(b) no

(c) yes

(d) yes

10-3 Relating the Linear and Angular Variables (7 of 13)

Designing The Giant Ring

We are given the job of designing a large horizontal ring that will rotate around a vertical axis and that will have a radius of $r = 33.1$ m (matching that of Beijing's The Great Observation Wheel, the largest Ferris wheel in the world). Passengers will enter through a door in the outer wall of the ring and then stand next to that wall (Fig. 10-10*a*). We decide that for the time interval $t = 0$ to $t = 2.30$ s, the angular position $\theta(t)$ of a reference line on the ring will be given by

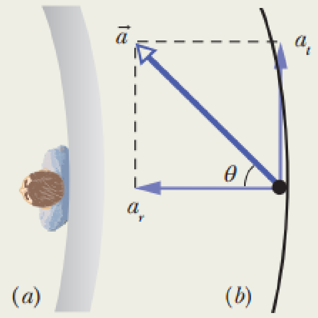
$$\theta = ct^3, \quad (10\text{-}24)$$

with $c = 6.39 \times 10^{-2}$ rad/s^3. After $t = 2.30$ s, the angular speed will be held constant until the end of the ride.

10-3 Relating the Linear and Angular Variables (8 of 13)

Designing The Giant Ring

Figure 10-10 (*a*) Overhead view of a passenger ready to ride The Giant Ring. (*b*) The radial and tangential acceleration components of the (full) acceleration.



10-3 Relating the Linear and Angular Variables (9 of 13)

Designing The Giant Ring

Once rotating, the floor of the ring will drop away from the riders but the riders will not fall—indeed, they feel as though they are pinned to the wall. For the time $t = 2.20$ s, let's determine a rider's angular speed ω, linear speed v, angular acceleration α, tangential acceleration a_t, radial acceleration a_r, and acceleration $\vec{a}$.

10-3 Relating the Linear and Angular Variables (10 of 13)

Designing The Giant Ring

Calculations: Let's go through the steps. We first find the angular velocity by taking the time derivative of the given angular position function and then substituting the given time of $t = 2.20$ s:

$$\omega = \frac{d\theta}{dt} = \frac{d}{dt}(ct^3) = 3ct^2 \qquad (10\text{-}25)$$

$$= 3(6.39 \times 10^{-2}\ \text{rad/s}^3)(2.20\ \text{s})^2$$

$$= 0.928\ \text{rad/s}. \qquad \text{(Answer)}$$

From Eq. 10-18, the linear speed just then is

$$v = \omega r = 3ct^2 r \qquad (10\text{-}26)$$

$$= 3(6.39 \times 10^{-2}\ \text{rad/s}^3)(2.20\ \text{s})^2(33.1\ \text{m})$$

$$= 30.7\ \text{m/s}. \qquad \text{(Answer)}$$

10-3 Relating the Linear and Angular Variables (11 of 13)

Designing The Giant Ring

Next, let's tackle the angular acceleration by taking the time derivative of Eq. 10-25:

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(3ct^2) = 6ct$$

$$= 6(6.39 \times 10^{-2}\ \text{rad/s}^3)(2.20\ \text{s}) = 0.843\ \text{rad/s}^2. \qquad \text{(Answer)}$$

The tangential acceleration then follows from Eq. 10-22:

$$a_t = \alpha r = 6ctr \qquad (10\text{-}27)$$

$$= 6(6.39 \times 10^{-2}\ \text{rad/s}^3)(2.20\ \text{s})(33.1\ \text{m})$$

$$= 27.91\ \text{m/s}^2 \approx 27.9\ \text{m/s}^2, \qquad \text{(Answer)}$$

10-3 Relating the Linear and Angular Variables (12 of 13)

Designing The Giant Ring

Substituting from Eq. 10-25 leads us to

$$a_r = \omega^2 r = (3ct^2)^2 r = 9c^2t^4 r \quad (10\text{-}28)$$

$$= 9(6.39 \times 10^{-2}\ \text{rad/s}^3)^2(2.20\ \text{s})^4(33.1\ \text{m})$$

$$= 28.49\ \text{m/s}^2 \approx 28.5\ \text{m/s}^2, \quad \text{(Answer)}$$

The radial and tangential accelerations are perpendicular to each other and form the components of the rider's acceleration $\vec{a}$ (Fig. 10-10*b*). The magnitude of $\vec{a}$ is given by

$$a = \sqrt{a_r^2 + a_t^2} \quad (10\text{-}29)$$

$$= \sqrt{(28.49\ \text{m/s}^2)^2 + (27.91\ \text{m/s}^2)^2}$$

$$\approx 39.9\ \text{m/s}^2, \quad \text{(Answer)}$$

10-3 Relating the Linear and Angular Variables (13 of 13)

Designing The Giant Ring

$$\theta = \tan^{-1}\left(\frac{6ctr}{9c^2t^4r}\right) = \tan^{-1}\left(\frac{2}{3ct^3}\right). \quad (10\text{-}30)$$

$$\theta = \tan^{-1}\frac{2}{3(6.39 \times 10^{-2}\ \text{rad/s}^3)(2.20\ \text{s})^3} = 44.4°. \quad \text{(Answer)}$$

10-4 Kinetic Energy of Rotation (1 of 8)

Learning Objectives

10.17 Find the rotational inertia of a particle about a point.

10.18 Find the total rotational inertia of many particles moving around the same fixed axis.

10.19 Calculate the rotational kinetic energy of a body in terms of its rotational inertia and its angular speed.

10-4 Kinetic Energy of Rotation (2 of 8)

- Apply the kinetic energy formula for a point particle and sum over all the particles

$$K = \sum \frac{1}{2} m_i v_i^2$$

different linear velocities (same angular velocity for all particles but possibly different radii)

- Then write velocity in terms of angular velocity:

$$K = \sum \frac{1}{2} m_i (\omega r_i)^2 = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2, \quad \textbf{Equation (10-32)}$$

We call the quantity in parentheses on the right side the **rotational inertia**, or **moment of inertia**, I

10-4 Kinetic Energy of Rotation (3 of 8)

- This is a constant for a rigid object and given rotational axis
- Caution: the axis for *I* must always be specified
- We can write:

$$I = \sum m_i r_i^2 \quad \text{(rotational inertia)}$$ **Equation (10-33)**

10-4 Kinetic Energy of Rotation (4 of 8)

- And rewrite the kinetic energy as:

$$K = \frac{1}{2} I\omega^2 \quad \text{(radian measure)}$$ **Equation (10-34)**

- Use these equations for a finite set of rotating particles
- Rotational inertia corresponds to how difficult it is to change the state of rotation (speed up, slow down or change the axis of rotation)

10-4 Kinetic Energy of Rotation (5 of 8)

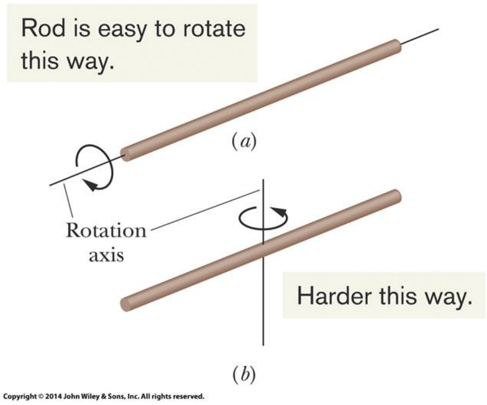
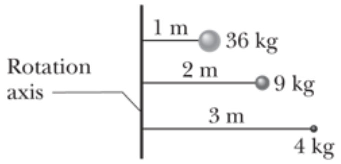


Figure 10-11

10-4 Kinetic Energy of Rotation (6 of 8)

Checkpoint 4

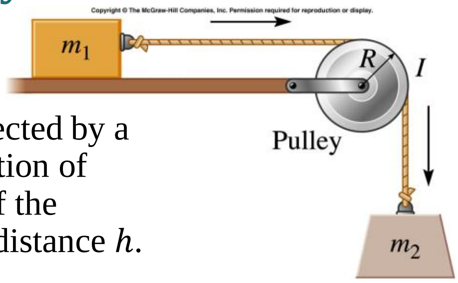
The figure shows three small spheres that rotate about a vertical axis. The perpendicular distance between the axis and the center of each sphere is given. Rank the three spheres according to their rotational inertia about that axis, greatest first.



Answer:

They are all equal!

10-4 Kinetic Energy of Rotation (7 of 8)



Example : Massless Pulley

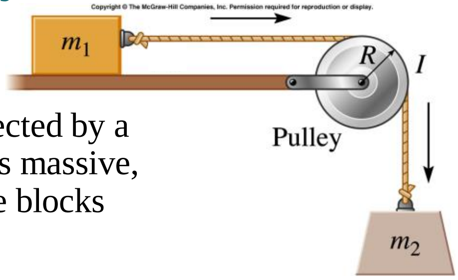
Consider the two masses connected by a pulley as shown. Use conservation of energy to calculate the speed of the blocks after m_2 has dropped a distance h. Assume the pulley is massless.

$$U_{initial} + K_{initial} = U_{final} + K_{final}$$

$$0 + 0 = -m_2gh + \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$$

$$2m_2gh = m_1v^2 + m_2v^2 \rightarrow v = \sqrt{\frac{2m_2gh}{m_1 + m_2}}$$

10-4 Kinetic Energy of Rotation (8 of 8)



Example : Massive Pulley

Consider the two masses connected by a pulley as shown. If the pulley is massive, after m_2 drops a distance h, the blocks will be moving

1) faster than
2) the same speed as
3) slower than

if it was a massless pulley?

Summary (1 of 7)

Angular Position

- Measured around a **rotation axis**, relative to a **reference line**:

$$\theta = \frac{s}{r}$$ **Equation (10-1)**

Angular Displacement

- A change in angular position

$$\Delta\theta = \theta_2 - \theta_1.$$ **Equation (10-4)**

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Summary (2 of 7)

Angular Velocity and Speed

- Average and instantaneous values:

$$\omega_{\text{avg}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t},$$ **Equation (10-5)**

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}.$$ **Equation (10-6)**

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Summary (3 of 7)

Angular Acceleration

- Average and instantaneous values:

$$\alpha_{\text{avg}} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}, \qquad \textbf{Equation (10-7)}$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}. \qquad \textbf{Equation (10-8)}$$

Summary (4 of 7)

Kinematic Equations

- Given in Table 10-1 for constant acceleration
- Match the linear case

Linear and Angular Variables Related

- Linear and angular displacement, velocity, and acceleration are related by *r*

Summary (5 of 7)

Rotational Kinetic Energy and Rotational Inertia

$$K = \tfrac{1}{2} I\omega^2 \quad \text{(radian measure)}$$ **Equation 10-34**

$$I = \sum m_i r_i^2 \quad \text{(rotational inertia)}$$ **Equation 10-33**

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