Fundamentals Physics

Eleventh Edition

Halliday

Chapter 10

Rotation

10-5 Calculating the Rotational Inertia (1 of 11)

Learning Objectives

10.20 Determine the rotational inertia of a body if it is given in Table 10-2.

10.21 Calculate the rotational inertia of body by integration over the mass elements of the body.

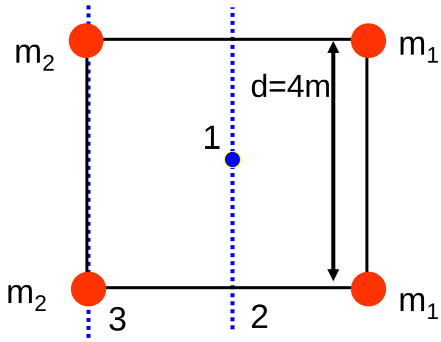
10.22 Apply the parallel-axis theorem for a rotation axis that is displaced from a parallel axis through the center of mass of a body.

Copyright ©2018 John Wiley & Sons, Inc

10-5 Calculating the Rotational Inertia (2 of 11)

- For discrete particles, we can write

$$I = \sum m_i r_i^2$$



- **Example**:
  Four masses are arranged in a square as shown (m_1 = 10 kg and m_2 = 20 kg). Find I for each of the given axes.

$$I_1 = 2m_1\left(\tfrac{d}{4}\sqrt{2}\right)^2 + 2m_2\left(\tfrac{d}{4}\sqrt{2}\right)^2$$
$$= 2(10\ kg)(2\ m^2) + 2(20\ kg)(2\ m^2) = 120\ kg\ m^2$$

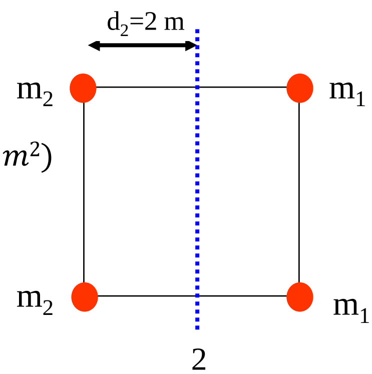
Copyright ©2018 John Wiley & Sons, Inc 3

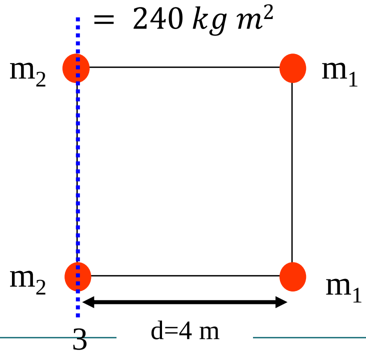
3

10-5 Calculating the Rotational Inertia (3 of 11)

The moment of inertia around axis 2 is:

$$I_2 = 2m_1 d_2^2 + 2m_2 d_2^2$$
$$= 2(10\ kg)(4\ m^2) + 2(20\ kg)(4\ m^2)$$
$$= 240\ kg\ m^2$$





The moment of inertia around axis 3 is:

$$I_3 = 2m_1 d^2 + 2m_2 \times 0$$
$$= 2(10\ kg)(4\ m)2 + 0 = 320\ kgm^2$$

Copyright ©2018 John Wiley & Sons, Inc 4

4

10-5 Calculating the Rotational Inertia (4 of 11)

- Integrating Equation.10-33 over a continuous body:

$$I = \int r^2 \, dm \quad \text{(rotational inertia, continuous body).}$$

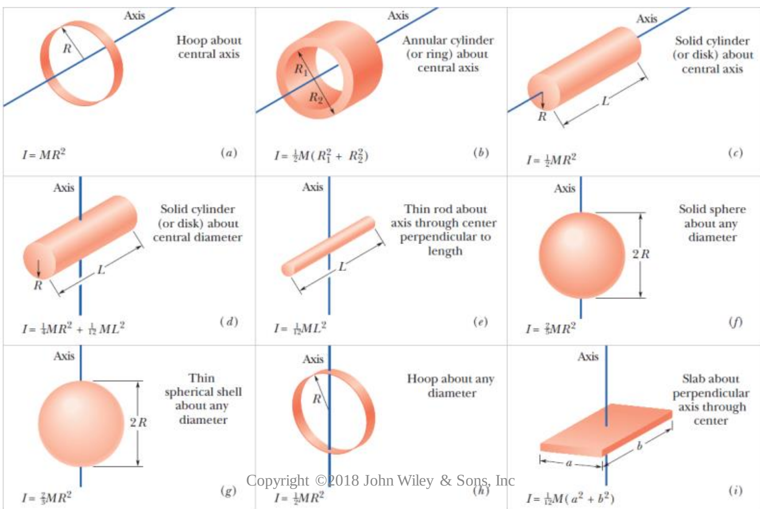
Equation (10-35)

- In principle we can always use this equation
- But there is a set of common shapes for which values have already been calculated (Table 10-2) for common axes

Copyright ©2018 John Wiley & Sons, Inc

10-5 Calculating the Rotational Inertia (5 of 11)

Table 10-2 Some Rotational Inertias



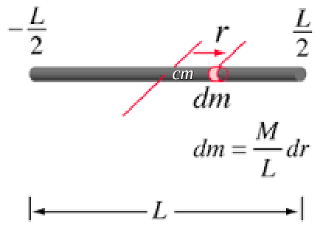
Copyright ©2018 John Wiley & Sons, Inc

10-5 Calculating the Rotational Inertia (6 of 11)

Example:

$$I_{cm} = \int_{-\frac{L}{2}}^{\frac{L}{2}} r^2 dm = \int_{-\frac{L}{2}}^{\frac{L}{2}} r^2 \frac{M}{L} dr$$

$$= \frac{M}{L}\frac{r^3}{3}\Bigg|_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{M}{3L}\left[\frac{L^3}{8} - \frac{-L^3}{8}\right] = \frac{1}{12}ML^2$$



Copyright ©2018 John Wiley & Sons, Inc

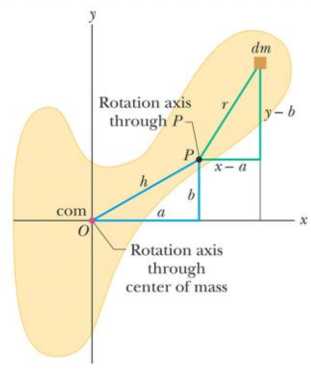
10-5 Calculating the Rotational Inertia (7 of 11)

- If we know the moment of inertia for the center of mass axis, we can find the moment of inertia for a parallel axis with the **parallel-axis theorem:**

  $I = I_{\text{com}} + Mh^2$ **Equation (10-36)**

- Note the axes must be parallel, and the first must go through the center of mass
- This does not relate the moment of inertia for two arbitrary axes

We need to relate the rotational inertia around the axis at *P* to that around the axis at the com.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

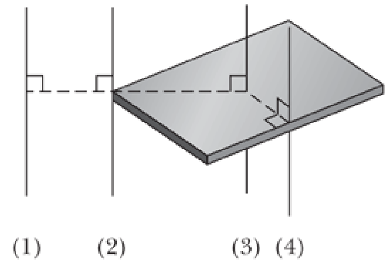
Figure 10-12

Copyright ©2018 John Wiley & Sons, Inc

10-5 Calculating the Rotational Inertia (8 of 11)

Checkpoint 5

The figure shows a book-like object (one side is longer than the other) and four choices of rotation axes, all perpendicular to the face of the object. Rank the choices according to the rotational inertia of the object about the axis, greatest first.



Answer:

(1), (2), (4), (3)

Copyright ©2018 John Wiley & Sons, Inc

10-5 Calculating the Rotational Inertia (9 of 11)

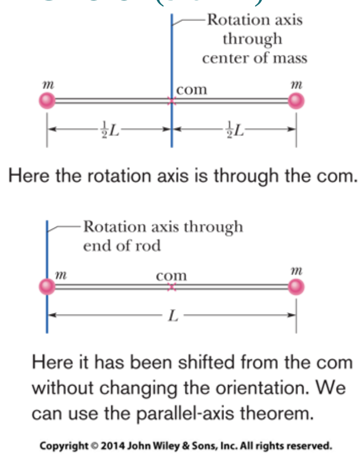


Figure 10-13

Example Calculate the moment of inertia for Figure. 10-13 (b)

- Summing by particle:

$$I = m(0)^2 + mL^2 = mL^2.$$

- Use the parallel-axis theorem

$$I = I_{com} + Mh^2$$
$$= \frac{1}{2}mL^2 + (2m)\left(\frac{1}{2}L\right)^2$$
$$= mL^2$$

Copyright ©2018 John Wiley & Sons, Inc

10-6 Torque (1 of 6)

Learning Objectives

10.23 Identify that a torque on a body involves a force and a position vector, which extends from a rotation axis to the point where the force is applied.

10.24 Calculate the torque by using (a) the angle between the position vector and the force vector, (b) the line of action and the moment arm of the force, and (c) the force component perpendicular to the position vector.

10.25 Identify that a rotation axis must always be specified to calculate a torque.

Copyright ©2018 John Wiley & Sons, Inc

10-6 Torque (2 of 6)

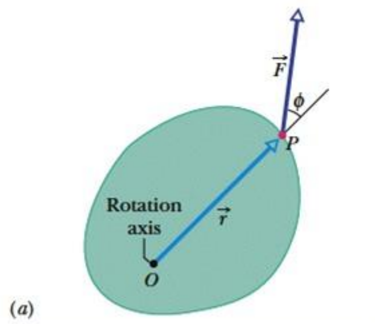
10.26 Identify that a torque is assigned a positive or negative sign depending on the direction it tends to make the body rotate about a specified rotation axis: "clocks are negative."

10.27 When more than one torque acts on a body about a rotation axis, calculate the net torque.

Copyright ©2018 John Wiley & Sons, Inc

10-6 Torque (3 of 6)

- The force necessary to rotate an object depends on the angle of the force and where it is applied
- We can resolve the force into components to see how it affects rotation



The torque due to this force causes rotation around this axis (which extends out toward you).

Figure 10-16

Copyright ©2018 John Wiley & Sons, Inc 13

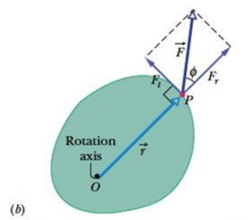
13

10-6 Torque (4 of 6)

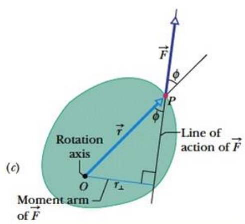
- **Torque** takes these factors into account:

$$\tau = (r)(F \sin \phi). \qquad \textbf{Equation (10-39)}$$

- A line extended through the applied force is called the **line of action** of the force
- The perpendicular distance from the line of action to the axis is called the **moment arm**
- The unit of torque is the newton-meter, N m
- Note that 1 J = 1 N m, but torques are never expressed in joules, torque is not energy



But actually only the *tangential* component of the force causes the rotation.



You calculate the same torque by using this moment arm distance and the full force magnitude.

Copyright ©2018 John Wiley & Sons, Inc 14

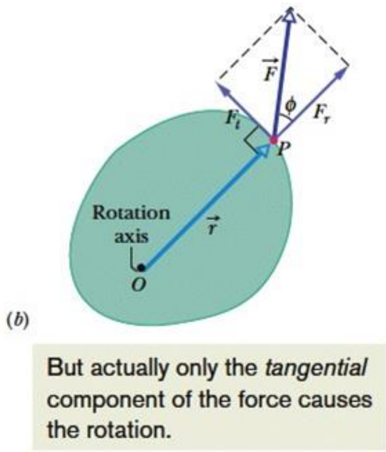
14

10-6 Torque (5 of 6)

- Again, torque is positive if it would cause a counterclockwise rotation, otherwise negative

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- For several torques, the **net torque** or **resultant torque** is the sum of individual torques



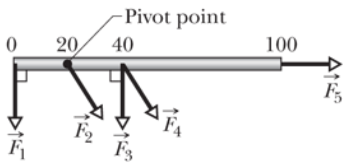
Copyright ©2018 John Wiley & Sons, Inc 15

15

10-6 Torque (6 of 6)

Checkpoint 6

The figure shows an overhead view of a meter stick that can pivot about the dot at the position marked 20 (for 20 cm). All five forces on the stick are horizontal and have the same magnitude. Rank the forces according to the magnitude of the torque they produce, greatest first.



Answer:

F_1 & F_3, F_4, F_2 & F_5

Copyright ©2018 John Wiley & Sons, Inc 16

16

10-7 Newton's Second Law for Rotation (1 of 8)

Learning Objectives

10.28 Apply Newton's second law for rotation to relate the net torque on a body to the body's rotational inertia and rotational acceleration, all calculated relative to a specified rotation axis.

Copyright ©2018 John Wiley & Sons, Inc 17

17

10-7 Newton's Second Law for Rotation (2 of 8)

- Rewrite $F = ma$ with rotational variables:

$$F_t = ma_t$$
$$F_t r = m(\alpha r)r$$

or,

$$\tau_{net} = I\alpha \qquad \textbf{Equation (10-42)}$$

- It is torque that causes angular acceleration

The torque due to the tangential component of the force causes an angular acceleration around the rotation axis.

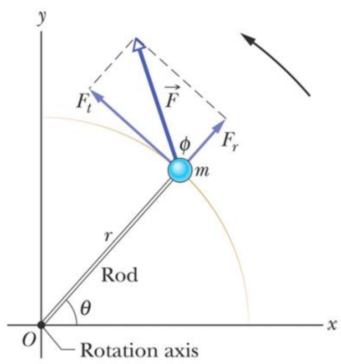


Figure 10-17

Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

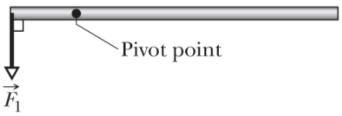
Copyright ©2018 John Wiley & Sons, Inc 18

18

10-7 Newton's Second Law for Rotation (3 of 8)

Checkpoint 7

The figure shows an overhead view of a meter stick that can pivot about the point indicated, which is to the left of the stick's midpoint. Two horizontal forces, $\vec{F}_1$ and $\vec{F}_2$, are applied to the stick. Only $\vec{F}_1$ is shown. Force $\vec{F}_2$ is perpendicular to the stick and is applied at the right end. If the stick is not to turn, (a) what should be the direction of $\vec{F}_2$, and (b) should F_2 be greater than, less than, or equal to F_1?



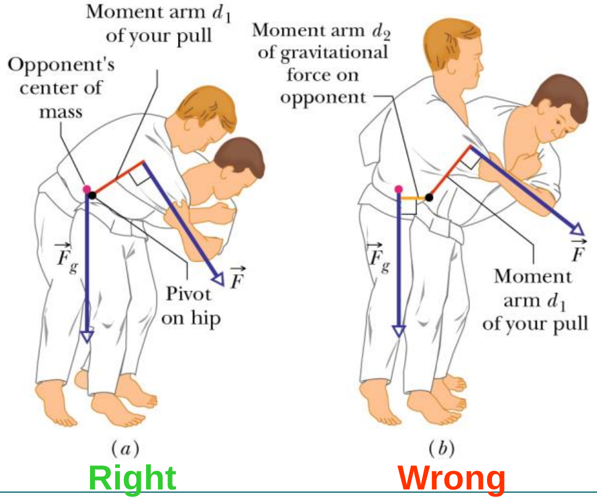
Answer:

(a) F_2 should point downward, and

(b) should have a smaller magnitude than F_1

Copyright ©2018 John Wiley & Sons, Inc

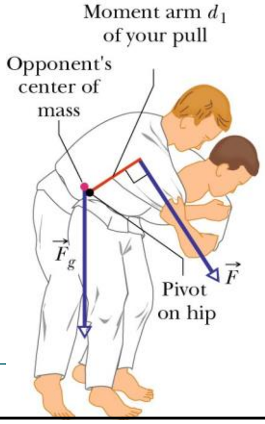
10-7 Newton's Second Law for Rotation (4 of 8)



Copyright ©2018 John Wiley & Sons, Inc

10-7 Newton's Second Law for Rotation (5 of 8)

What must $\vec{F}$ be for a ***good*** throw, if $d_1 = 0.3$ m and assume that your angular acceleration is $\alpha = -6$ rad/s^2 and the moment of inertia of your opponent is 15 kg m^2



Only force $\vec{F}$ that resulted in a torque, thus

$$\tau_{net} = I\alpha$$

or

$$-d_1 F = I\alpha$$

yielding

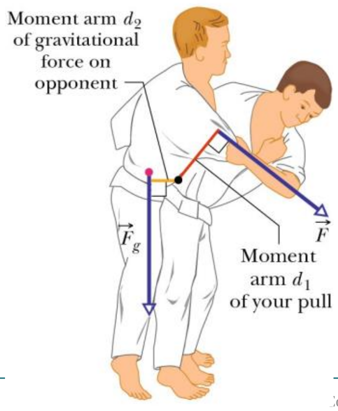
$$F = \frac{-I\alpha}{d_1} = \frac{-(15)(-6.0)}{0.3} = 300\ N$$

Copyright ©2018 John Wiley & Sons, Inc 21

21

10-7 Newton's Second Law for Rotation (6 of 8)

If the opponent remains upright, then you will have a bad throw. What must $\vec{F}$ be, if $d_2 = 0.12$ m ?



The net torque resulted from the force $\vec{F}$ and the gravitational force $\vec{F}_g$

$$\tau_{net} = I\alpha \quad \text{or} \quad -d_1 F + d_2 mg = I\alpha$$

yielding

$$F = \frac{-I\alpha}{d_1} + \frac{d_2 mg}{d_1}$$

$$= 300 + \frac{(0.12)(80)(9.8)}{0.3}$$

$$= 613.6\ N$$

Copyright ©2018 John Wiley & Sons, Inc 22

22

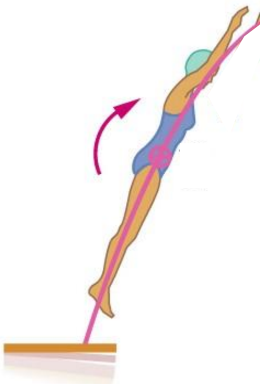
10-7 Newton's Second Law for Rotation (7 of 8)

Example

During a launch from the board, a diver's angular speed about her center of mass changes from zero to 6.20 rad/s in 220 ms. Her rotational inertia about her center of mass is 12.0 kg m^2. During the launch, what are the magnitudes of

a) her average angular acceleration and

b) the average external torque on her from the board?



Copyright ©2018 John Wiley & Sons, Inc

10-7 Newton's Second Law for Rotation (8 of 8)

a) Average angular acceleration

$$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{6.20\,\frac{\text{rad}}{\text{s}} - 0\,\frac{rad}{s}}{0.220\text{ s}}$$

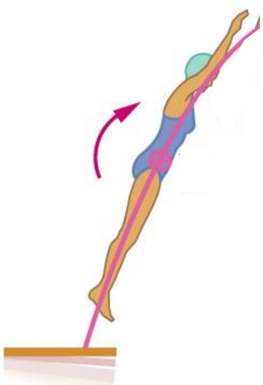
$$= 28.2\,\frac{\text{rad}}{\text{s}^2}$$

b) Average external torque

$$\tau = I\alpha$$

$$= (12.0\text{ kg m}^2)\left(28.2\,\frac{\text{rad}}{\text{s}^2}\right)$$

$$= 338\text{ Nm}$$



Copyright ©2018 John Wiley & Sons, Inc

10-8 Work and Rotational Kinetic Energy (1 of 9)

Learning Objectives

10.29 Calculate the work done by a torque acting on a rotating body by integrating the torque with respect to the angle of rotation.

10.30 Apply the work-kinetic energy theorem to relate the work done by a torque to the resulting change in the rotational kinetic energy of the body.

10.31 Calculate the work done by a constant torque by relating the work to the angle through which the body rotates.

10.32 Calculate the power of a torque by finding the rate at which work is done.

10.33 Calculate the power of a torque at any given instant by relating it to the torque and the angular velocity at that instant.

Copyright ©2018 John Wiley & Sons, Inc

10-8 Work and Rotational Kinetic Energy (2 of 9)

- The rotational work-kinetic energy theorem states:

$$\Delta K = K_f - K_i = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2 = W$$ **Equation (10-52)**

- The work done in a rotation about a fixed axis can be calculated by:

$$W = \int F ds = \int F r d\theta = \int_{\theta_i}^{\theta_f} \tau d\theta$$ **Equation (10-53)**

- Which, for a constant torque, reduces to:

$$W = \tau(\theta_f - \theta_i)$$ **Equation (10-54)**

Copyright ©2018 John Wiley & Sons, Inc

10-8 Work and Rotational Kinetic Energy (3 of 9)

- We can relate work to power with the equation:

$$P = \frac{dW}{dt} = \tau\omega$$ **Equation (10-55)**

- Table 10-3 shows corresponding quantities for linear and rotational motion:

Copyright ©2018 John Wiley & Sons, Inc 27

27

10-8 Work and Rotational Kinetic Energy (4 of 9)

Table 10-3 Some Corresponding Relations for Translational and Rotational Motion

Pure Translation (Fixed Direction)		**Pure Rotation (Fixed Axis)**	
Position	x	Angular position	θ
Velocity	$v = \frac{dx}{dt}$	Angular velocity	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	Angular acceleration	$\alpha = \frac{d\omega}{dt}$
Mass	m	Rotational inertia	I
Newton's second law	$F_{net} = ma$	Newton's second law	$\tau_{net} = I\alpha$
Work	$W = \int F\, dx$	Work	$W = \int \tau\, d\theta$

Copyright ©2018 John Wiley & Sons, Inc 28

28

10-8 Work and Rotational Kinetic Energy (5 of 9)

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

Copyright © 2018 John Wiley & Sons, Inc. All rights reserved.

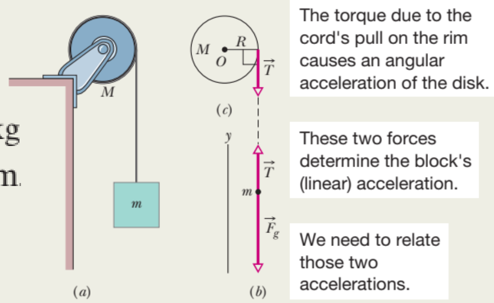
10-8 Work and Rotational Kinetic Energy (6 of 9)

Work, rotational kinetic energy, torque, disk

Let the disk in Fig. 10-19 start from rest at time $t = 0$ and also let the tension in the massless cord be 6.0 N and the angular acceleration of the disk be $-24\ \text{rad/s}^2$. What is its rotational kinetic energy K at $t = 2.5$ s?

$M = 2.5$ kg

$R = 20$ cm



10-8 Work and Rotational Kinetic Energy (7 of 9)

Work, rotational kinetic energy, torque, disk

Calculations: Because we want ω and know α and ω_0 $(=0)$, we use Eq. 10-12:

$$\omega = \omega_0 + \alpha t = 0 + \alpha t = \alpha t.$$

Substituting $\omega = \alpha t$ and $I = \frac{1}{2}MR^2$ into Eq. 10-34, we find

$$\begin{aligned} K &= \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(\tfrac{1}{2}MR^2)(\alpha t)^2 = \tfrac{1}{4}M(R\alpha t)^2 \\ &= \tfrac{1}{4}(2.5\text{ kg})[(0.20\text{ m})(-24\text{ rad/s}^2)(2.5\text{ s})]^2 \\ &= 90\text{ J}. \qquad \text{(Answer)} \end{aligned}$$

Copyright ©2018 John Wiley & Sons, Inc

10-8 Work and Rotational Kinetic Energy (8 of 9)

Work, rotational kinetic energy, torque, disk

We can also get this answer by finding the disk's kinetic energy from the work done on the disk.

$$W = \tau(\theta_f - \theta_i) = -TR(\theta_f - \theta_i). \qquad (10\text{-}61)$$

Because α is constant, we can use Eq. 10-13 to find $\theta_f - \theta_i$. With $\omega_i = 0$, we have

$$\theta_f - \theta_i = \omega_i t + \tfrac{1}{2}\alpha t^2 = 0 + \tfrac{1}{2}\alpha t^2 = \tfrac{1}{2}\alpha t^2.$$

Copyright ©2018 John Wiley & Sons, Inc

10-8 Work and Rotational Kinetic Energy (9 of 9)

Work, rotational kinetic energy, torque, disk

Now we substitute this into Eq. 10-61 and then substitute the result into Eq. 10-60. Inserting the given values $T = 6.0$ N and $\alpha = -24$ rad/s^2, we have

$$K = W = -TR(\theta_f - \theta_i) = -TR(\tfrac{1}{2}\alpha t^2) = -\tfrac{1}{2}TR\alpha t^2$$
$$= -\tfrac{1}{2}(6.0\text{ N})(0.20\text{ m})(-24\text{ rad/s}^2)(2.5\text{ s})^2$$
$$= 90\text{ J}. \qquad \text{(Answer)}$$

Copyright ©2018 John Wiley & Sons, Inc

Summary (1 of 7)

Angular Position

- Measured around a **rotation axis**, relative to a **reference line**:

$$\theta = \frac{s}{r} \qquad \textbf{Equation (10-1)}$$

Angular Displacement

- A change in angular position

$$\Delta\theta = \theta_2 - \theta_1. \qquad \textbf{Equation (10-4)}$$

Copyright ©2018 John Wiley & Sons, Inc

Summary (2 of 7)

Angular Velocity and Speed

- Average and instantaneous values:

$$\omega_{\text{avg}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}, \qquad \textbf{Equation (10-5)}$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}. \qquad \textbf{Equation (10-6)}$$

Copyright ©2018 John Wiley & Sons, Inc

Summary (3 of 7)

Angular Acceleration

- Average and instantaneous values:

$$\alpha_{\text{avg}} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}, \qquad \textbf{Equation (10-7)}$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}. \qquad \textbf{Equation (10-8)}$$

Copyright ©2018 John Wiley & Sons, Inc

Summary (4 of 7)

Kinematic Equations

- Given in Table 10-1 for constant acceleration
- Match the linear case

Linear and Angular Variables Related

- Linear and angular displacement, velocity, and acceleration are related by *r*

Copyright ©2018 John Wiley & Sons, Inc

Summary (5 of 7)

Rotational Kinetic Energy and Rotational Inertia

$$K = \tfrac{1}{2} I\omega^2 \quad \text{(radian measure)}$$ **Equation 10-34**

$$I = \sum m_i r_i^2 \quad \text{(rotational inertia)}$$ **Equation 10-33**

The Parallel-Axis Theorem

- Relate moment of inertia around any parallel axis to value around com axis

$$I = I_{\text{com}} + Mh^2$$ **Equation 10-36**

Copyright ©2018 John Wiley & Sons, Inc

Summary (6 of 7)

Torque

- Force applied at distance from an axis:

$$\tau = (r)(F \sin \phi).$$ **Equation (10-39)**

- Moment arm: perpendicular distance to the rotation axis

Newton's Second Law in Angular Form

$$\tau_{\text{net}} = I\alpha$$ **Equation (10-42)**

Copyright ©2018 John Wiley & Sons, Inc

Summary (7 of 7)

Work and Rotational Kinetic Energy

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta$$ **Equation (10-53)**

$$P = \frac{dW}{dt} = \tau\omega$$ **Equation (10-55)**

Copyright ©2018 John Wiley & Sons, Inc

Copyright

Copyright © 2018 John Wiley & Sons, Inc.

All rights reserved. Reproduction or translation of this work beyond that permitted in Section 117 of the 1976 United States Act without the express written permission of the copyright owner is unlawful. Request for further information should be addressed to the Permissions Department, John Wiley & Sons, Inc. The purchaser may make back-up copies for his/her own use only and not for distribution or resale. The Publisher assumes no responsibility for errors, omissions, or damages, caused by the use of these programs or from the use of the information contained herein.