Fundamentals Physics

Eleventh Edition

Halliday

Chapter 19

The Kinetic Theory of Gases

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What we have learnt

- The 1st Law of Thermodynamics

$$\Delta E_{int} = Q - W \quad \text{or} \quad Q = \Delta E_{int} + W$$

- Work is defined as

$$W = \int_{V_i}^{V_f} p\, dV$$

- For processes where the pressure is not constant what is p as a function of V ?
- Further, we have said that Internal Energy depended on temperature,

$$\Delta E_{int} \propto T$$

but what is the exact expression ?

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19-1 Avogadro's Number (1 of 3)

Learning Objectives

19.01 Identify Avogadro's number N_A.

19.02 Apply the relationship between the number of moles n, the number of molecules N, and Avogadro's number N_A.

19.03 Apply the relationships between the mass m of a sample, the molar mass M of the molecules in the sample, the number of moles n in the sample, and Avogadro's number N_A.

19-1 Avogadro's Number (2 of 3)

The **kinetic theory of gases** relates the macroscopic properties of gases to the microscopic properties of gas molecules.

One **mole** of a substance contains N_A (**Avogadro's number**) elementary units (usually atoms or molecules), where N_A is found experimentally to be $N_A = 6.02 \times 10^{23} \text{mol}^{-1}$.

The mass per mole M of a substance is related to the mass m of an individual molecule of the substance by $M = mN_\text{A}$.

19-1 Avogadro's Number (3 of 3)

The number of moles n contained in a sample of mass M_{sam}, consisting of N molecules, is related to the molar mass M of the molecules and to Avogadro's number N_A as given by

$$n = \frac{M_{sam}}{M} = \frac{M_{sam}}{mN_A}.$$

19-2 Ideal Gases (2 of 7)

19.08 Sketch p-V diagrams for a constant-temperature expansion of a gas and a constant-temperature contraction.

19.09 Identify the term isotherm.

19.10 Calculate the work done by a gas, including the algebraic sign, for an expansion and a contraction along an isotherm.

19-2 Ideal Gases (3 of 7)

19.11 For an isothermal process, identify that the change in internal energy ΔE is zero and that the energy Q transferred as heat is equal to the work W done.

19.12 On a p-V diagram, sketch a constant-volume process and identify the amount of work done in terms of area on the diagram.

19.13 On a p-V diagram, sketch a constant-pressure process and determine the work done in terms of area on the diagram.

19-2 Ideal Gases (4 of 7)

An ideal gas is one for which the pressure p, volume V, and temperature T are related by

$$pV = nRT$$

Here n is the number of moles of the gas present and R is a constant (8.31 J/mol.K) called the gas constant.

The Second Expression for the law is:

$$pV = NkT$$

where k is the Boltzmann constant

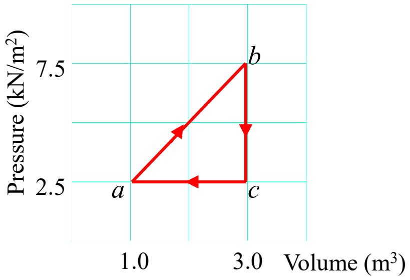
19-2 Ideal Gases (4 of 7)

Example A Cycle

A sample of an ideal gas is taken through the cyclic process *abca* shown in the figure; at point a, T = 200 K. (a) How many moles of gas are in the sample? What are (b) the temperature of the gas at point *b*, (c) the temperature of the gas at point *c*, and (d) the net heat added to the gas during the cycle?

(a) $n = \dfrac{p_A V_A}{R T_A} = 1.5\text{ mol.}$

(b) $T_B = \dfrac{p_B V_B}{nR} = 1.8\times\ 10^3\text{ K}$



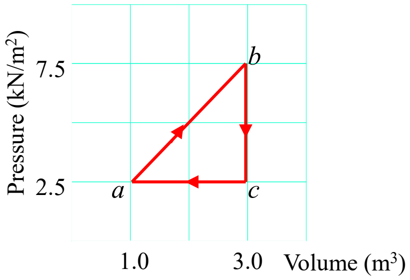
19-2 Ideal Gases (4 of 7)

Example A Cycle

(c) $T_C = \dfrac{p_C V_C}{nR} = 6.0\times\ 10^2\text{ K}$

(d) Cyclic process $\Rightarrow \Delta E_{int} = 0$

$Q = W =$ Enclosed Area= 0.5 x 2m^3 x 5x10^{3}Pa = 5.0 x 10^3 J

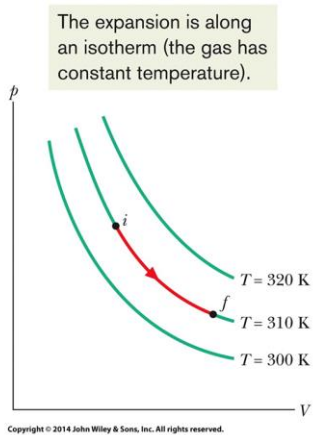


19-2 Ideal Gases (6 of 7)

The work done by an ideal gas during an isothermal (constant-temperature) change from volume V_i to volume V_f is

$$W = nRT \ln \frac{V_f}{V_i}$$

Three isotherms on a *p*-*V* diagram. The path shown along the middle isotherm represents an isothermal expansion of a gas from an initial state *i* to a final state *f*. The path from *f* to *i* along the isotherm would represent the reverse process – that is, an isothermal compression.



19-3 Pressure, Temperature, and rms Speed (1 of 4)

Learning Objectives

19.14 Identify that the pressure on the interior walls of a gas container is due to the molecular collisions with the walls.

19.15 Relate the pressure on a container wall to the momentum of the gas molecules and the time intervals between their collisions with the wall.

19.16 For the molecules of an ideal gas, relate the root-mean-square speed v_{rms} and the average speed v_{avg}.

19-3 Pressure, Temperature, and rms Speed (2 of 4)

19.17 Relate the pressure of an ideal gas to the rms speed v_{rms} of the molecules.

19.18 For an ideal gas, apply the relationship between the gas temperature T and the rms speed v_{rms} and molar mass M of the molecules.

19-3 Pressure, Temperature, and rms Speed (3 of 4)

a microscopic picture

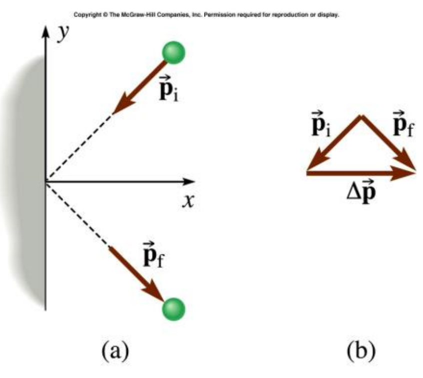
Pressure results from collisions of molecules on the surface

$$P = \frac{F}{A}$$

Force:

$$F = \frac{dp}{dt}$$

Rate of momentum given to the surface

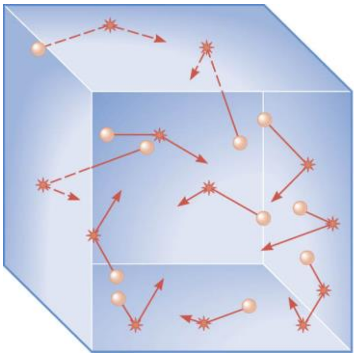


19-3 Pressure, Temperature, and rms Speed (3 of 4)

a microscopic picture

Consider N number of particles in a volume V

Assume that all particles have the same velocity : v_x



19-3 Pressure, Temperature, and rms Speed (3 of 4)

a microscopic picture

The number of collisions hitting an area A in time dt is

$$\frac{1}{2}\left(\frac{N}{V}\right)Av_x dt$$

N is the number of particles
V is the volume

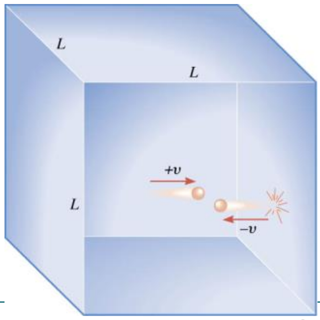
2 directions | Average density

The momentum given by each collision to the surface

$$2mv_x$$

Momentum in time dt:

$$dp = (2mv_x)\frac{1}{2}\left(\frac{N}{V}\right)Av_x dt$$



19-3 Pressure, Temperature, and rms Speed (3 of 4)

$$dp = (2mv_x)\frac{1}{2}\left(\frac{N}{V}\right)Av_x dt$$

a microscopic picture

Force:

$$F = \frac{dp}{dt} = (2mv_x)\frac{1}{2}\left(\frac{N}{V}\right)Av_x$$

Pressure:

$$p = \frac{F}{A} = \frac{N}{V}mv_x^2$$

Not all molecules have the same v_x^2 $\Rightarrow$ average $\overline{v_x^2}$

$$p = \frac{N}{V}m\overline{v_x^2} = \frac{1}{3}\frac{N}{V}mv_{rms}^2$$

v_{rms} is the root-mean-square speed $v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}}$

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19-3 Pressure, Temperature, and rms Speed (3 of 4)

In terms of the speed of the gas molecules, the pressure exerted by *n* moles of an ideal gas is

$$p = \frac{1}{3}\frac{N}{V}mv_{rms}^2 = \frac{nMv_{\text{rms}}^2}{3V},$$

where v_{rms} is the root-mean-square speed of the molecules, *M* is the molar mass, and *V* is the volume.

For an ideal gas, the rms speed can be written in terms of the temperature as

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}.$$

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19-3 Pressure, Temperature, and rms Speed (4 of 4)

Table 19-1 Some RMS Speeds at Room Temperature (T = 300 K)[a]

Gas	Molar Mass $(10^{-3}\,\text{kg/mol})$	v_{rms} (m/s)
Hydrogen (H_2)	2.02	1920
Helium (He)	4.0	1370
Water vapor (H_2O)	18.0	645
Nitrogen (N_2)	28.0	517
Oxygen (O_2)	32.0	483
Carbon dioxide (CO_2)	44.0	412
Sulfur dioxide (SO_2)	64.1	342

[a]For convenience, we often set room temperature equal to 300 K even though (at 27°C or 81°F) that represents a fairly warm room.

19-4 Translational Kinetic Energy (1 of 3)

Learning Objectives

19.19 For an ideal gas, relate the average kinetic energy of the molecules to their rms speed.

19.20 Apply the relationship between the average kinetic energy and the temperature of the gas.

19.21 Identify that a measurement of a gas temperature is effectively a measurement of the average kinetic energy of the gas molecules.

19-4 Translational Kinetic Energy (1 of 3)

$$\overline{K} = \frac{1}{2}m\overline{v^2} = \frac{1}{2}mv_{rms}^2$$

$$P = \frac{1}{3}\frac{N}{V}mv_{rms}^2$$

Pressure:

$$P = \frac{2}{3}\cdot\frac{N}{V}\cdot\overline{K}$$

$$P = \frac{nRT}{V}$$

Temperature:

$$\overline{K} = \frac{3}{2}\cdot\frac{nRT}{N} = \frac{3}{2}\cdot k_B T$$

$$n = \frac{N}{N_A}$$

Boltzmann constant: $k_B = \frac{R}{N_A} = 1.38\times10^{-23}$ J/K

19-4 Translational Kinetic Energy (2 of 3)

The **average translational kinetic energy** is related to the temperature of the gas:

$$K_{avg} = \frac{3}{2}k_B T.$$

At a given temperature T, all ideal gas molecules—no matter what their mass—have the same average translational kinetic energy—namely, $\frac{3}{2}kT$. When we measure the temperature of a gas, we are also measuring the average translational kinetic energy of its molecules.

19-4 Translational Kinetic Energy (2 of 3)

Internal Energy

For monatomic gas: the internal energy = sum of the kinetic energy of all molecules:

$$E_{\text{int}} = N \cdot \overline{K} = nN_A \cdot \frac{3}{2} k_B T = \frac{3}{2} nRT$$

$$E_{\text{int}} = \frac{3}{2} nRT = \frac{3}{2} Nk_B T \propto T$$

19-4 Translational Kinetic Energy (2 of 3)

Equipartition of Energy

The internal energy of non-monatomic molecules includes also vibrational and rotational energies besides the translational energy.

The average translational kinetic energy is

$$\bar{K} = \frac{3}{2} k_B T$$

Monatomic gases have 3 translational degrees of freedom

Each degree of freedom has associated with an energy

of $\frac{1}{2} k_B T$ per molecules.

19-4 Translational Kinetic Energy (2 of 3)

Equipartition of Energy

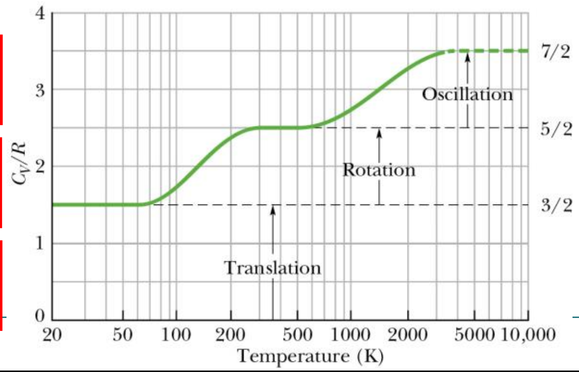
Diatomic Gases
(not all mode excited)

- 3 translational degrees of freedom
- 2 rotational degrees of freedom
- 2 vibrational degrees of freedom

$$E_{\text{int}} = \frac{7}{2} nRT$$

$$E_{\text{int}} = \frac{5}{2} nRT$$

$$E_{\text{int}} = \frac{3}{2} nRT$$



19-6 The Distribution of Molecular Speed (1 of 4)

Learning Objectives

19.24 Explain how Maxwell's speed distribution law is used to find the fraction of molecules with speeds in a certain speed range.

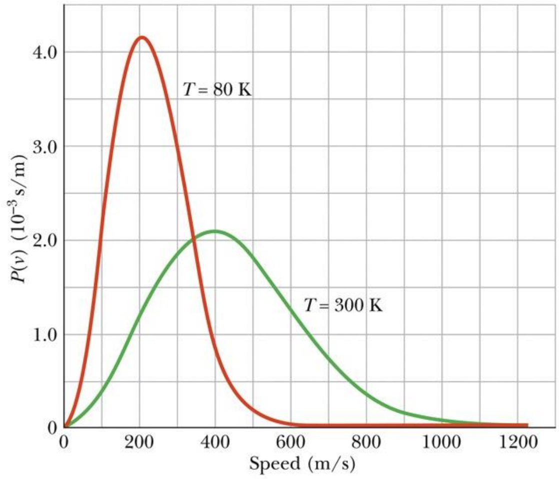
19.25 Sketch a graph of Maxwell's speed distribution, showing the probability distribution versus speed and indicating the relative positions of the average speed v_{avg}, the most probable speed v_P, and the rms speed v_{rms}.

19-6 The Distribution of Molecular Speed (2 of 4)

19.26 Explain how Maxwell's speed distribution is used to find the average speed, the rms speed, and the most probable speed.

19.27 For a given temperature T and molar mass M, calculate the average speed v_{avg}, the most probable speed v_P, and the rms speed v_{rms}.

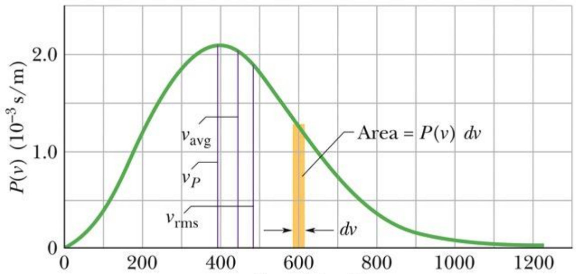
19-6 The Distribution of Molecular Speed (3 of 4)



19-6 The Distribution of Molecular Speed (3 of 4)

The **Maxwell speed distribution** $P(v)$ is a function such that $P(v)dv$ gives the fraction of molecules with speeds in the interval dv at speed v:

$$P(v) = 4\pi\left(\frac{M}{2\pi RT}\right)^{\frac{3}{2}} v^2 e^{\frac{-Mv^2}{2RT}}.$$



(a)

19-6 The Distribution of Molecular Speed (4 of 4)

Three measures of the distribution of speeds among the molecules of a gas:

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}} \quad \text{(average speed)},$$

$$v_P = \sqrt{\frac{2RT}{M}} \quad \text{(most probable speed)},$$

$$v_{rms} = \sqrt{\frac{3RT}{M}} \quad \text{(rms speed)}.$$

19-7 Molar Specific Heats of an Ideal Gas

(1 of 15)

Learning Objectives

19.28 Identify that the internal energy of an ideal monatomic gas is the sum of the translational kinetic energies of its atoms.

19.29 Apply the relationship between the internal energy E_{int} of a monatomic ideal gas, the number of moles n, and the gas temperature T.

19.30 Distinguish between monatomic, diatomic, and polyatomic ideal gases.

19-7 Molar Specific Heats of an Ideal Gas

(2 of 15)

19.31 For monatomic, diatomic, and polyatomic ideal gases, evaluate the molar specific heats for a constant-volume process and a constant-pressure process.

19.32 Calculate a molar specific heat at constant pressure C_p by adding R to the molar specific heat at constant volume C_V, and explain why (physically) C_p is greater.

19-7 Molar Specific Heats of an Ideal Gas

(3 of 15)

19.33 Identify that the energy transferred to an ideal gas as heat in a constant-volume process goes entirely into the internal energy but that in a constant-pressure process energy also goes into the work done to expand the gas.

19.34 Identify that for a given change in temperature, the change in the internal energy of an ideal gas is the same for any process and is most easily calculated by assuming a constant-volume process.

19-7 Molar Specific Heats of an Ideal Gas

(4 of 15)

19.35 For an ideal gas, apply the relationship between heat Q, number of moles n, and temperature change ΔT, using the appropriate molar specific heat.

19.36 Between two isotherms on a p-V diagram, sketch a constant-volume process and a constant-pressure process, and for each identify the work done in terms of area on the graph.

19-7 Molar Specific Heats of an Ideal Gas

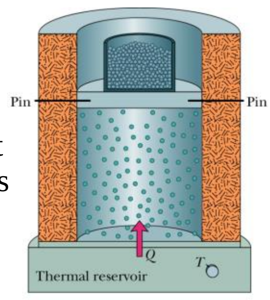
(5 of 15)

19.37 Calculate the work done by an ideal gas for a constant-pressure process.

19.38 Identify that work is zero for constant volume.

19-7 Molar Specific Heats of an Ideal Gas

Heat Transfer to an Ideal Gas at Constant Volume



(a)

The amount of heat transferred from the reservoir to the system at a process with constant volume (isovolumetric, isokhor) is given in terms of its *molar specific heat*, C_V

$$Q = nC_V\Delta T \longleftarrow Q = mc\Delta T$$

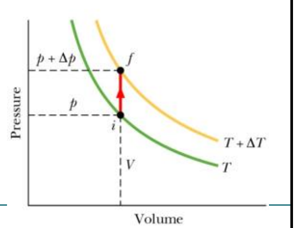
Because the volume is fixed → $W = 0$

Hence from the First Law we have,

$$\Delta E_{int} = Q - W = Q = nC_V\Delta T$$

For a monatomic gas, the internal energy given as :

$$\Delta E_{int} = \frac{3}{2}nR\Delta T \rightarrow C_V = \frac{3}{2}R$$



(b)

19-7 Molar Specific Heats of an Ideal Gas

Heat Transfer to an Ideal Gas at Constant Pressure

At constant pressure (isobaric) process, the amount of heat transferred to the system is given in terms of its *molar specific heat,* C_P

$$Q = nC_P\Delta T$$

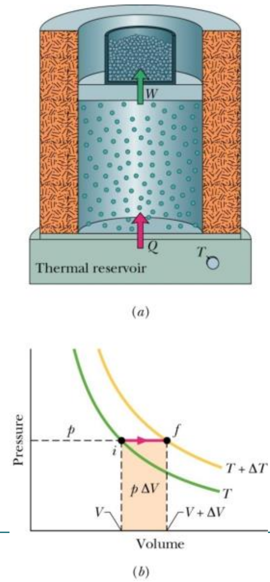
The work done is,

$$W = \int_{V_i}^{V_f} p dV = p(V_f - V_i) = p\Delta V = nR\Delta T$$

For a monatomic gas,

$$\Delta E_{\text{int}} = Q - W \rightarrow \frac{3}{2}nR\Delta T = nC_p\Delta T - nR\Delta T$$

$$C_P = C_V + R = \frac{5}{2}R$$



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19-9 Adiabatic Expansion of an Ideal Gas (1 of 4)

Learning Objectives

19.44 On a *p-V* diagram, sketch an adiabatic expansion (or contraction) and identify that there is no heat exchange *Q* with the environment.

19.45 Identify that in an adiabatic expansion, the gas does work on the environment, decreasing the gas's internal energy, and that in an adiabatic contraction, work is done on the gas, increasing the internal energy.

19.46 In an adiabatic expansion or contraction, relate the initial pressure and volume to the final.

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19-9 Adiabatic Expansion of an Ideal Gas (2 of 4)

19.47 In an adiabatic expansion or contraction, relate the initial temperature and volume to the final temperature and volume.

19.48 Calculate the work done in an adiabatic process by integrating the pressure with respect to volume.

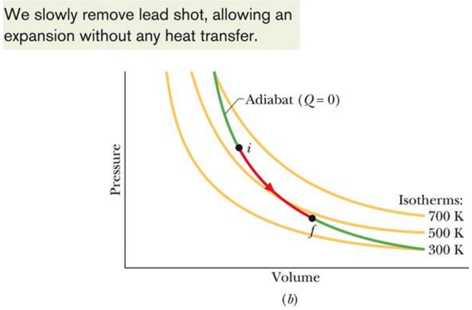
19.49 Identify that a free expansion of a gas into a vacuum is adiabatic but no work is done and thus, by the first law of thermodynamics, the internal energy and temperature of the gas do not change.

19-9 Adiabatic Expansion of an Ideal Gas

When an ideal gas undergoes a slow adiabatic volume change (a change for which $Q = 0$),

$$pV^{\gamma} = \text{a constant} \quad (\text{adiabatic process}),$$

in which $\gamma\left(=\frac{C_p}{C_V}\right)$ is the ratio of molar specific heats for the gas.



(a) The volume of an ideal gas is increased by removing mass from the piston. The process is adiabatic (Q= 0).

(b) The process proceeds from i to f along an adiabat on a p-V diagram.

Summary (1 of 5)

Kinetic Theory of Gases

- relates the macroscopic properties of gases to the microscopic properties of gas molecules.

Avogadro's Number

$$N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$$ **Equation (19-1)**

- Mole related to mass of a molecule

$$M = mN_A$$ **Equation (19-4)**

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Summary (2 of 5)

Ideal Gas

An ideal gas is one for which the pressure p, volume V, and temperature T are related by

$$pV = nRT \quad (\text{ideal gas law}).$$ **Equation (19-5)**

Temperature and Kinetic Energy

- The average translational kinetic energy per molecule of an ideal gas is

$$K_{\text{avg}} = \tfrac{3}{2}KT.$$ **Equation (19-24)**

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Summary (3 of 5)

Maxwell Speed Distribution

- The three measures of distribution of speed

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}} \quad \text{(average speed)},$$ **Equation (19-31)**

$$v_P = \sqrt{\frac{2RT}{M}} \quad \text{(most probable speed)},$$ **Equation (19-35)**

$$v_{rms} = \sqrt{\frac{3RT}{M}} \quad \text{(rms speed)}.$$ **Equation (19-22)**

Summary (4 of 5)

Molar Specific Heat

- At constant volume:

$$C_V = \frac{Q}{n\Delta T} = \frac{\Delta E_{int}}{n\Delta T}$$ **Equation (19-39&41)**

Degrees of Freedom

- The equipartition of energy theorem states that every degree of freedom of a molecule has an energy $\frac{1}{2}kT$ per molecule.

Summary (5 of 5)

Adiabatic Expansion

- When an ideal gas undergoes an adiabatic volume change (a change for which $Q = 0$)

$$pV^{\gamma} = \text{a constact}$$ **Equation (19-31)**

in which $\gamma = \left(= \frac{C_p}{C_V}\right)$ is the ratio of molar specific heats for the gas.

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